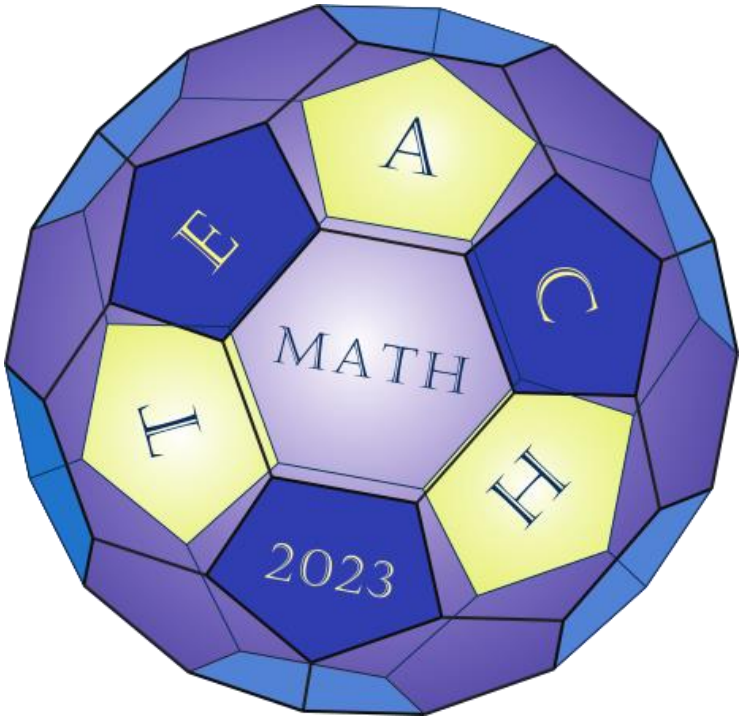
Josip Juraj Strossmayer University of Osijek

Faculty of Education **Department of Mathematics**



The 9th International Scientific Colloquium
MATHEMATICS AND CHILDREN
founded by Margita Pavleković

PROGRAM
&
BOOK OF ABSTRACTS

Editors:

Zdenka Kolar-Begović
Ružica Kolar-Šuper
Ana Katalenić

Croatia, Osijek, May 19-20, 2023

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PROGRAM

FRIDAY, May 19

8.00 - 9.00 Registration

9.00 - 9.30 **OPENING**

Morning Session

9.30 - 10.15	**Plenary lecture** Milena Stavric, Albert Wiltsche *Spatial ability in the digital age - designing innovative analogue games*
10.15 - 10.30	Attila Bölcskei *Connecting statistics with spatial abilities*
10.30 - 10.45	András Ambrus, Pál Katonka *"Seeing" at solving algebraical problems*
10.45 - 11.00	Ljiljana Arambašić *On the teaching of linear algebra*

11.00 - 11.30 Coffee Break

11.30 - 11.45	Doris Dumičić Danilović, Sanja Rukavina *On the use of visualization when performing basic arithmetic operations*
11.45 - 12.00	Ana Krišto, Ana Kuzle, Dubravka Glasnović Gracin *Collective classroom climate in geometry lessons*
12.00 - 12.15	Janka Szeibert, Csilla Zámbó, Anna Muzsnay, Csaba Szabó *The potential of number theory in the development of mathematical thinking*
12.15 - 12.30	Adrijana Mastnak, Tatjana Hodnik *Preschool teacher's planning and implementation of dialogic teaching in dealing with combinatorics situations*
12.30 - 12.45	Branka Antunović Piton *Implementation of scientific findings in the 7th-grade teaching process - Preliminary results of collaborative action research*
12.45 - 13.00	Sanja Vranić *Fermi problems as a mathematical modeling activity in secondary education*

13.00 - 14.00 Lunch

Afternoon Session

14.00 - 14.45	**Plenary lecture** Jasmina Milinković *Representations in mathematics teaching - How to make a choice?*
14.45 - 15.00	Vida Manfreda Kolar, Klara Paulič, Uršula Podobnik *The role of mathematical picture books in teaching the concept of zero to first grade*
15.00 - 15.15	Eszter Kónya, Ibolya Szilágyiné-Szinger *On the experience of a problem-posing activity with second grade primary school pupils*
15.15 - 15.30	Coffee Break

A section

15.30 - 15.45	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar *Development of computational thinking through interdisciplinary activities in mathematics education*
15.45 - 16.00	Tomislav Rudec, Anja Šteko *Advanced education methods and application of mathematical knowledge in classes with advanced programmers*
16.00 - 16.15	Saša Duka, Ružica Kolar-Šuper, Zdenka Kolar-Begović *The specifics of solving mathematical problems using model based on the Tablet-Human Hybrid Model of Avatar in university face-to-face teaching*
16.15 - 16.30	Lidija Eret, Marjana Kuliš *Hybrid teaching and outcome indicators of the state graduation exam in mathematics*

B section

15.30 - 15.45	Marijana Zeljić, Milana Dabić Boričić, Svetlana Ilić *Superficial strategies in solving compare-combine problems*
15.45 - 16.00	Sanja Rukavina, Doris Dumičić Danilović, Marina Šimac *Mathematics teachers' continuous professional development - Reflections on Lifelong Learning Program "Enactive Learning in Mathematics"*
16.00 - 16.15	Hana Čadež, Ana Brudar *The importance of the mathematics teacher and special educator working together to support low-achieving pupils in mathematics*

16.15 - 16.30	Edith Debrenti, Tünde Klára Baranyai *Analysis of logical operational abilities and logical reasoning among university students*

16.45 Excursion and Colloquium Dinner (Erdut, Stari mlin)

SATURDAY, May 20

Morning Session

9.00 - 9.45	**Plenary lecture** Mirela Jukić Bokun, Ljerka Jukić Matić *Game-based learning in mathematics*
9.45 - 10.00	Eleonóra Stettner *STEAM-BOX project*
10.00 - 10.15	Monika Zupančić, Tatjana Hodnik *The traditional–contemporary construct in relation to the learning paradigms in the discipline of didactics of mathematics*
10.15 - 10.30	Josipa Čuka, Maja Cindrić, Maja Čuletić Čondrić *Teacher's beliefs about teaching mathematics and teaching practice in Croatian primary schools*

10.30 - 11.00 Coffee Break

A section

11.00 - 11.30	Emil Molnár *On Kárteszi points of a triangle, via three reflections theorem and geometric algebra*
11.30 - 11.45	Maja Čuletić Čondrić, Maja Cindrić, Josipa Čuka *The importance of thought transformations in learning geometry based on Van Hiele's theory*
11.45 - 12.00	Nikolina Kovačević *Learner-generated drawings in mathematics: Who? When? How?*
12.00 - 12.15	Marija Jakuš *Sudoku puzzle*

12.15 - 12.30	Maja Cindrić, Josipa Čuka, Maja Čuletić Čondrić *Mathematics teaching in public schools in Croatian countries in the era of proto-modernization*

B section

11.00 - 11.15	Zoran Horvat, Ružica Jurčević *Digital pedagogy and teaching mathematics - Trends, perspectives, limitations and challenges*
11.15 - 11.30	Ivana Đurđević Babić, Natalija Bošnjaković *Decision trees in research on mathematics in elementary school*
11.30 - 11.45	Dubravka Glasnović Gracin, Ana Krišto *Requirements in digital mathematics textbooks: Development of the framework for task analysis*
11.45 - 12.00	Anja Horvat, Hana Horvat, Hanna Jakovac, Anamarija Kanisek, Ana Katalenić *Word problems in the textbooks for primary mathematics education in Croatia*
12.00 - 12.15	Goran Trupčević, Anđa Valent, Ana Žgaljić Keko *Discipline of noticing – The case of three university teachers*
12.15 - 12.30	Darija Marković, Ljerka Jukić Matić *Games in mathematics classroom: What do teachers and students think about them?*
12.30 - 12.45	Ljiljanka Kvesić, Maja Lasić, Marina Zubac *Mathematical teaching of preschool children through elements of performing arts*

13.00 CLOSING

LUNCH

Contents

Plenary lectures

Contributed talks

ABSTRACTS

Game-based learning in mathematics

Mirela Jukić Bokun and Ljerka Jukić Matić

Department of Mathematics, Josip Juraj Strossmayer University of Osijek, Croatia

Abstract. Mathematics is often viewed as an abstract body of knowledge that is disconnected from the real world, unlike other STEM disciplines. Thus, students may experience a sense of disconnection from the mathematical topics. Additionally, traditional teaching methods are criticized for their inability to engage students and for denying them the autonomy to construct their own comprehension. Due to its interactive nature, Game-Based Learning (GBL), particularly Digital Game-Based Learning (DGBL), appears to be a promising approach to learning and teaching in Mathematics Education. Digital games can foster a safe environment in which Socio-Scientific Issues (SSI) are discussed, and students may be more motivated to learn not only mathematics but also the societal issues that relate to Mathematics Education in the real world. Increasing students' motivation to learn is another reason why teachers should implement DGBL in the classroom. Numerous digital games, for instance, require mastery of strategic and analytical thinking, problem-solving, and decision-making, all of which are valuable in the modern workforce. Students of the twenty-first century benefit greatly from digital games for this reason. In addition, computer games have unique characteristics (such as fantasy, rules/goals, sensory stimuli, challenge, mystery, and control) that can increase learning motivation and result in attitude and behaviour change. In contemporary education, DGBL is an effective medium for students to learn mathematical concepts and practice mathematical skills. Using and implementing DGBL is a challenge in education, and teachers play a central role in overcoming this obstacle. In the DGBL method, teachers are designers, facilitators, or guides. Therefore, they must be assisted in acquiring the knowledge and skills necessary to assume these roles.

We will introduce the Erasmus+ project GAMMA (GAMe-based learning in MAthematics) by discussing the essential aspects of DGBL: DGBL definition,

learning and teaching mathematics with DGBL, designing games for teaching mathematics, and applying games to mathematics education. To maximize the benefits of technology and DGBL, the GAMMA project aims to use the time that students already spend on their electronic devices and their knowledge and experience of digital games to enhance their mathematical knowledge and skills.

Keywords: DGBL, digital games, mathematics teachers, GAMMA project

Učenje matematike temeljeno na (digitalnim) igrama

Mirela Jukić Bokun i Ljerka Jukić Matić

Odjel za matematiku, Sveučilište Josipa Jurja Strossmayera u Osijeku, Hrvatska

Sažetak. Matematika se često smatra apstraktnom znanosti, koja nije povezana sa stvarnim svijetom za razliku od drugih STEM disciplina. Radi toga se učenici često ne mogu saživjeti s matematičkim temama. Zbog svoje interaktivne prirode, učenje temeljeno na igrama (GBL), posebno učenje temeljeno na digitalnim igrama (DGBL), čini se obećavajućim pristupom učenju i poučavanju u nastavi matematike. Digitalne igre mogu potaknuti sigurno okruženje u kojem se raspravlja o društveno-znanstvenim pitanjima, a učenici mogu biti značajno motivirani za učenje ne samo matematike već i društvenih pitanja koja se odnose na matematičko obrazovanje u stvarnom svijetu. Povećanje motivacije učenika za učenje još je jedan razlog zašto bi učitelji trebali implementirati DGBL u razred. Brojne digitalne igre zahtijevaju ovladavanje strateškim i analitičkim razmišljanjem, rješavanjem problema i donošenjem odluka, što je sve od izrazite važnosti za mnoge poslove, stoga učenici dvadeset prvog stoljeća imaju velike koristi od digitalnih igara. Osim toga, karakteristike računalnih igara (kao što su fantazija, pravila/ciljevi, osjetilni podražaji, izazov, misterija i kontrola) mogu rezultirati promjenom stava i ponašanja. Stoga je u suvremenom obrazovanju DGBL učinkovit medij za učenje matematičkih pojmova i izgradnju matematičkih vještina. Korištenje i implementacija DGBL-a predstavlja izazov u nastavi, a nastavnici igraju središnju ulogu u prevladavanju tog izazova. U DGBL metodi učitelji su dizajneri, voditelji ili vodiči. Stoga im se mora pomoći u stjecanju znanja i vještina potrebnih za preuzimanje ovih uloga.

Predstavit ćemo Erasmus+ projekt GAMMA (GAMe-based learning in MAthematics) raspravljajući o bitnim aspektima DGBL-a: definiciji DGBL-a, učenju i poučavanju matematike pomoću DGBL-a, dizajniranju igara za podučavanje matematike i primjeni igara u matematičkom obrazovanju. Kako bi se iskoristile prednosti tehnologije i DGBL-a, projekt GAMMA želi iskoristiti vrijeme koje

učenici već provode na svojim elektroničkim uređajima te primijeniti njihovo znanje i iskustvo u digitalnim igrama kako bi unaprijedili svoje matematičko znanje i vještine.

Ključne riječi: DGBL, digitalne igre, nastavnici matematike, GAMMA projekt

Representations in mathematics teaching – How to make a choice?

Jasmina Milinković

Teacher Education Faculty, University of Belgrade, Serbia

Abstract. Representations are one of the key terms in modern didactics of mathematics at all levels of education. The significance of representations in the formation of mathematical concepts will be discussed. Next, representations in the process of designing mathematical problems or making a choice of tasks for students will be explored. Finally, the role of representations in the process of solving problems is explored. In the address, we will touch upon all these aspects of using representations with the intention to justify why they are considered crucial in learning mathematics. In particular, we will consider the role of teachers in the use of representations in teaching mathematics.

Keywords: mathematical concepts, representations, problem solving, problem posing

Reprezentacije u nastavi matematike – Kako napraviti izbor?

Jasmina Milinković

Učiteljski fakultet, Univerzitet u Beogradu, Srbija

Sažetak. Reprezentacije su jedan od ključnih pojmova u savremenoj didaktici matematike na svim nivoima obrazovanja. Biće reči o značaju reprezentacija u formiranju matematičkih pojmova. Zatim će se istražiti reprezentacije u procesu dizajniranja matematičkih zadataka ili izbora zadataka za učenike. Konačno, biće istražena uloga reprezentacija u procesu rešavanja problema. U obraćanju ćemo se dotaknuti svih ovih aspekata korišćenja reprezentacija sa namerom da opravdamo zašto se one smatraju ključnim u učenju matematike. Posebno ćemo razmotriti ulogu nastavnika u korišćenju reprezentacija u nastavi matematike.

Ključne reči: matematički pojmovi, reprezentacije, rešavanje problema, postavljanje problema

Spatial ability in the digital age – designing innovative analogue games

Milena Stavric and Albert Wiltsche

Institute of Architecture and Media, Graz University of Technology, Austria

Abstract. How can we support and develop the spatial ability of young people in order to increase creativity and curiosity? Can we connect creativity with the mathematical and geometrical way of thinking? In our talk, we will share some of our experiences in developing innovative analogue games, making prototypes, and implementing such games in the education curriculum.

Keywords: spatial ability, creativity, mathematics, geometry, games

Raumvorstellung im digitalen Zeitalter – Gestaltung innovativer analoger Spiele

Milena Stavric und Albert Wiltsche

Institut für Architektur und Medien, Technische Universität Graz, Austria

Abstrakt. Wie können wir das räumliche Vorstellungsvermögen junger Menschen fördern und entwickeln, um ihre Kreativität und Neugierde zu steigern? Können wir Kreativität mit der mathematischen und geometrischen Denkweise verbinden? In unserem Vortrag werden wir einige unserer Erfahrungen mit der Entwicklung innovativer analoger Spiele, der Herstellung von Prototypen und der Implementierung solcher Spiele in den Lehrplan vorstellen.

Schlüsselwörter: Raumvorstellung, Kreativität, Mathematik, Geometrie, Spiele

"Seeing" at solving algebraical problems

András Ambrus[1] and Pál Katonka[2]

[1]Institute of Mathematics, Eötvös Lóránd University, Hungary
[2]Institute of Mathematics, University of Debrecen, Hungary

Abstract. NCTM defines mathematical connections in Principals and Standards for School Mathematics as the ability to "recognise and use connections among mathematical ideas; understand how mathematical ideas are connected and build on each other to form a coherent whole; recognise and apply mathematics in contexts outside mathematics". This is a fine goal, but what is the school reality? We will analyse a student's solution of a parametric square root equation where his algebraic solution was incomplete. Discussion with this student convinced him of the usefulness of the geometric solution. Various topics are interlinked here: functions, equations, analytical geometry, and calculus. No wonder students have difficulties in solving such problems. In our presentation, we will give a brief theoretical background: Baddeley memory system, working memory, phonological loop, visuospatial buffer, dual code theory, representations, schemas, and concept map, showing their relevance for solving our problem. Finally, we formulate some conclusions for mathematics teaching.

Keywords: algebra, parametric equations, representations, mathematical connections, schema, concept map

"Látni" az algebrai problémák megoldásánál

András Ambrus[1] és Pál Katonka[2]

[1]Matematikai Intézet, Eötvös Loránd Tudományegyetem, Magyarország
[2]Matematikai Intézet, Debreceni Egyetem, Magyarország

Absztrakt. Az NCTM a matematikai kapcsolatokat úgy határozza meg, mint a "matematikai elvek, ismeretek közötti kapcsolatok felismerésének és használatának képességét; megérteni, hogy a matematikai ismeretek hogyan kapcsolódnak egymáshoz, és hogyan épülnek egymásra, hogy koherens egészet hozzanak létre; felismerni és alkalmazni a matematikát a matematikán kívüli összefüggésekben". Szép cél, de mi az iskolai valóság? Előadásunkban elemezzük egy tanuló paraméteres négyzetgyökös egyenletre adott megoldását, hiányos algebrai megoldást adott. A tanulóval folytatott beszélgetés meggyőzte őt a geometriai megoldás hasznosságáról. Itt különböző témák kapcsolódnak egymáshoz: függvények, egyenletek, analitikus geometria, differenciálszámítás, nem csoda, hogy a tanulóknak nehézségei adódhatnak az ilyen jellegű feladatok megoldásában. Előadásunkban rövid elméleti hátteret adunk: Baddeley memóriarendszer, munkamemória, fonológiai hurok, vizuális térbeli puffer, kettős kód elmélet, reprezentációk, sémák, fogalomtérkép, bemutatva ezek relevanciáját a probléma megoldásában. Végül megfogalmazunk néhány következtetést a matematikatanítással kapcsolatban.

Kulcsszavaki: algebra, parametrikus egyenletek, reprezentációk, matematikai összefüggések, séma, fogalomtérk

Implementation of scientific findings in the 7th-grade teaching process – Preliminary results of collaborative action research

Branka Antunović Piton

Faculty of Educational Sciences, Juraj Dobrila University of Pula, Croatia

Abstract. The paper presents research that aims to bridge the gap between theory and practice in mathematics education by implementing research findings into mathematics teaching through collaboration between the academic community and schools. Following the researcher's program, two mathematics teachers taught polygons and circles to 7th-grade students, utilizing the latest research results. The teaching strategy focused on connecting various representations of the same content, decomposing geometric figures into their constituent parts, and operating with figure elements to promote deeper understanding and achieve better learning outcomes in geometry. Initially, the researcher played a dominant role in the program, but through the process of action research, the teachers took on a more prominent role and continued to collaborate with the researcher, resulting in higher-quality teaching and better learning outcomes. The proposed model of action research demonstrates a more productive and natural way of applying new knowledge in teaching practice. Preliminary research results and an overview of the motivational factors that influence student participation will be presented. Moreover, the research findings suggest positive changes in teachers' attitudes towards teaching geometry and fostering a new culture in which teachers take responsibility for applying action research in their classrooms to transform the learning process.

Keywords: action research, teacher's attitude, collaboration, geometrical thinking

Implementacija znanstvenih spoznaja u nastavni proces 7. razreda – Preliminarni rezultati kolaboracijskog akcijskog istraživanja

Branka Antunović Piton

Fakultet za odgojne i obrazovne znanosti, Sveučilište Jurja Dobrile u Puli, Hrvatska

Sažetak. Rad predstavlja istraživanje koje ima za cilj premostiti jaz između teorije i prakse u matematičkom obrazovanju implementacijom rezultata istraživanja u nastavu matematike kroz suradnju akademske zajednice i škola. Slijedeći program istraživača, dvije učiteljice matematike poučavale su učenike 7. razreda o mnogokutima, kružnici i krugu, koristeći najnovije rezultate istraživanja. Strategija poučavanja bila je usmjerena na povezivanje različitih prikaza istog sadržaja, dekonstruiranje geometrijskih likova na njihove sastavne dijelove i rad s elementima figure kako bi se promicalo dublje razumijevanje i postigli bolji rezultati učenja u geometriji. U početku je istraživač imao dominantnu ulogu u programu, ali kroz proces akcijskog istraživanja učiteljice su preuzele sve istaknutiju ulogu i nastavile surađivati s istraživačem, što je rezultiralo kvalitetnijim poučavanjem i boljim ishodima učenja. Predloženi model akcijskog istraživanja pokazuje produktivniji i prirodniji način primjene novih znanja u nastavnoj praksi. U ovom radu prikazuju se preliminarni rezultati istraživanja te pregled nekih čimbenika koji utječu na motivaciju učenika u testiranju. Nadalje, nalazi istraživanja ukazuju na pozitivne promjene u stavovima nastavnika prema poučavanju geometrije i njegovanju nove kulture u kojoj učitelji preuzimaju odgovornost za primjenu akcijskog istraživanja u svojim učionicama kako bi transformirali proces učenja.

Ključne riječi: akcijsko istraživanje, stavovi učitelja, kolaboracija, geometrijsko mišljenje

On the teaching of linear algebra

Ljiljana Arambašić

Department of Mathematics, Faculty of Science, University of Zagreb, Croatia

Abstract. Linear algebra courses are a standard part of the first year of mathematics studies. For most students, it is their first encounter with an abstract and conceptual way of studying mathematics, and some of them have difficulties with understanding linear algebra. Some of these difficulties are a large amount of new definitions for which they see neither sense nor motivation, complicated proofs whose main ideas need to be memorized, and the disconnection of linear algebra with the knowledge of mathematics acquired in previous education. In order to overcome these difficulties, teachers pay special attention to the way the material is presented, and in this lecture, we will show several examples that we have implemented in our own teaching practice.

Keywords: abstract concepts, linear algebra, proofs, mathematics education, higher education

O nastavi linearne algebre

Ljiljana Arambašić

Matematički odsjek, Prirodoslovno-matematički fakultet, Sveučilište u Zagrebu, Hrvatska

Sažetak. Na prvoj godini matematičkih studija se izvode kolegiji iz linearne algebre. Za većinu studenata to je prvi susret s apstraktnim i konceptualnim načinom proučavanja matematike, te dio njih ima poteškoće s razumijevanjem linearne algebre. Neke od tih poteškoća su velika količina novih definicija za koje ne vide ni smisao ni motivaciju, komplicirani dokazi čije je glavne ideje potrebno memorirati, te nepovezanost linearne algebre sa znanjem matematike stečenim u prethodnom obrazovanju. Kako bi premostili ove poteškoće, nastavnici posebnu pažnju posvećuju načinu prezentiranja gradiva, a u ovom predavanju navest ćemo nekoliko primjera koje smo implementirali u vlastitoj nastavnoj praksi.

Ključne riječi: apstraktni koncepti, linearna algebra, dokazi, poučavanje matematike, visoko obrazovanje

Connecting statistics with spatial abilities

Attila Bölcskei

Department of Methodology for Social Studies, Faculty of International Management and Business, Budapest Business School, University of Applied Sciences, Hungary

Abstract. Our earlier research showed that the teaching of spatial geometry has been significantly reduced in the Hungarian education system in recent decades. Similar tendencies are also well-known internationally. Thus, one of the key elements that could be useful in developing spatial intelligence was reduced in mathematics education. This is problematic because many research proved that the development of spatial abilities has a positive effect on the completion of science and STEM subjects, so it would help to reach the coveted reduction of drop-out rates. In addition, many studies argue that spatial abilities should also play a greater role in teaching of e.g. geography, GIS, sociology, medicine, business sciences and marketing. On the other hand, research proves that the improvement of spatial intelligence is of importance in therapies of dyslexia and dyscalculia.

In contrast to all these, there is a trend that is contrary to the above: increased expansion of statistics content in mathematics teaching can be observed. Taking advantage of this fact, the presentation focuses on how statistical tasks could be used to develop spatial thinking. We make suggestions on the types of this kind of tasks and analyze how they would be suitable for developing spatial intelligence factors, especially those of mental rotation and visualization.

Keywords: teaching statistics, spatial intelligence, visualization, mental rotation, reducing drop-out rates

A statisztika és térlátás összekapcsolása

Attila Bölcskei

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Absztrakt. Korábbi kutatásaink bebizonyították, hogy a magyar oktatási rendszerben az elmúlt évtizedekben a térgeometria oktatása jelentősen visszaszorult. Ez a folyamat nemzetközi viszonylatban is jól ismert. A matematika oktatásában tehát a téri intelligenciát fejlesztő egyik legfontosabb elem redukálódott. Mindez azért is sajnálatos, mert a téri képességek fejlesztése bizonyított módon pozitív hatással van a természettudományos és STEM tárgyak elvégzésére, így a lemorzsolódás áhított csökkentése irányában hatna. Ezen túl számos tanulmány érvel amellett, hogy a téri képességeknek nagyobb szerepet kéne kapnia a földrajz, térinformatika, szociológia, orvostudomány, üzleti tudományok és marketing oktatása során is. Emellett kutatások igazolják, hogy a téri intelligencia javításával például sikerek érhetők el még a diszlexia ill. a diszkalkulia terápiájában is.

Mindezekkel szemben a statisztika iskolai oktatásával kapcsolatban a fentiekkel ellentétes tendencia, annak fokozott térnyerése figyelhető meg. Ezért a jelen előadásban azt vizsgáljuk, hogy milyen módon lehetne a statisztikai feladatokat egyúttal a térszemlélet fejlesztésére felhasználni. Javaslatokat fogalmazunk meg azzal kapcsolatban, hogy milyen feladattípusok és milyen módon lennének alkalmasak a téri intelligencia faktorainak fejlesztésére. Ezen belül külön foglalkozunk két képességelemnek, a mentális forgatásnak és a vizualizációnak a lehetséges fejlesztésével.

Kulcsszavak: statisztika oktatása, téri intelligencia, vizualizáció, mentális forgatás, lemorzsolódás csökkentése

Mathematics teaching in public schools in Croatian countries in the era of proto-modernization

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Abstract. The foundations of modern secular education were laid on the basis of the enlightenment movements in the second half of the 18th century in the territory of the Croatian lands, then part of the Habsburg Monarchy. The reform of lower education lasted until the first half of the 19th century and was based on the structure of the personnel and legislative documents. This paper provides an overview of the changes in the approach to teaching basic mathematical content, the characteristics of which have remained until today, based on the textbook literature of that era. The textbooks were analyzed from the aspect of the methodical analysis of content, goals, the terminological and semantic structure of mathematical concepts, and the classification of tasks based on the Task Type and Mathematics Learning (TTML) classification. The methodical analysis includes a comparison of approaches and goals before and after the school reform of 1777., and in particular the contribution of Franjo Močnik in the methodical design of arithmetics teaching in the public schools of that time, as well as of Franjo Klaić, according to whose methodical guidelines the teaching of mathematics in lower grades continues even today. The terminological analysis provides insight into the development and ambiguity of the terminology of basic mathematical concepts and the misconceptions they entail. The paper will also present a detailed analysis of the classification of tasks in four representative arithmetics textbooks from the era of proto-modernization.

Keywords: arithmetics, school reform, Franjo Močnik, Franjo Klaić

Nastava matematike u pučkim školama na području hrvatskih zemalja u doba protomodernizacije

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Sažetak. Temeljem prosvjetiteljskih pokreta u drugoj polovici 18. stoljeća na području hrvatskih zemalja, tada u sklopu Habsburške monarhije, postavljaju se temelji modernog sekularnog školstva. Reforma nižeg školstva traje sve do prve polovice 19. stoljeća, a bazira se na ustroju kadrovske strukture i zakonodavnih akata. Ovaj rad donosi pregled promjena u pristupu poučavanja osnovnih matematičkih sadržaja čije odlike su se zadržale do danas, temeljem udžbeničke literature tog doba. Udžbenici su analizirani s aspekta metodičke raščlambe sadržaja, ciljeva i očekivanja, terminološke i semantičke strukture matematičkih pojmova, te klasifikacije zadataka temeljem TTML klasifikacije. Metodička raščlamba uključuje usporedbu pristupa i ciljeva prije i nakon školske reforme 1777. godine, te posebice doprinos Franje Močnika u metodičkom oblikovanju nastave računice u tadašnjim pučkim školama, kao i Franje Klaića prema čijim metodičkim smjernicama se odvija nastava matematike u nižim razredima i danas. Terminološka analiza daje uvid u razvoj i dvojbenost terminologije osnovnih matematičkih pojmova, te miskoncepcija koje povlače. Rad će prikazati i detaljnu analizu klasifikacije zadataka u četiri reprezentativna udžbenika računice iz doba protomodernizacije.

Ključne riječi: računica, školska reforma, Franjo Močnik, Franjo Klaić

Development of computational thinking through interdisciplinary activities in mathematics education

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Abstract. By using digital tools in mathematics classes, we can achieve many positive effects: increase students' motivation and active participation in the teaching process, develop a computational way of thinking, achieve a higher level of knowledge and a better understanding of mathematical concepts through active learning, introduce problem-based learning, develop students' digital literacy, positive attitude towards mathematics and an awareness of the importance of mathematics. We will present some activities which we can implement in mathematics classes to achieve the goals that we have set. In preparation for these activities, we have intensively used digital tools, especially 3D modelling and 3D printing. If the conditions are met, it is very often possible to involve students in the preparation of certain activities using digital tools.

Keywords: computational thinking, ICT tools, interdisciplinary approach, problem-based learning, mathematics

Razvoj računalnog mišljenja kroz interdisciplinarne aktivnosti u matematičkom obrazovanju

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Sažetak. Upotrebom digitalnih alata u nastavi matematike dolazimo do mnogih pozitivnih učinaka: povećavamo motivaciju učenika i aktivno sudjelovanje u nastavnom procesu, razvijamo računalni način razmišljanja, aktivnim učenjem postižemo veću razinu znanja i bolje razumijevanje matematičkih koncepata, uvodimo učenje temeljeno na problemima, razvijamo digitalnu pismenost učenika, pozitivan stav prema matematici i svijest o značaju matematike. Prezentirat ćemo primjere aktivnosti koje možemo realizirati u nastavi matematike kako bi postigli zadane ciljeve. Prilikom pripreme aktivnosti intenzivno smo koristili digitalne alate, posebno 3D modeliranje i 3D print. Za navedene aktivnosti vrlo često je moguće, ako za to postoje uvjeti, u pripremu pojedine aktivnosti uz pomoć digitalnih alata uključiti i učenike.

Ključne riječi: računalni način razmišljanja, IKT alati, interdisciplinarni pristup, učenje temeljeno na problemima, matematika

The importance of the mathematics teacher and special educator working together to support low-achieving pupils in mathematics

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Abstract. The number of pupils receiving intensive and specific additional support in learning mathematics is increasing every year. Ensuring quality support for lower-achieving pupils is a growing concern in mathematics education, including the issue of how to provide competent professionals for intensive and specific additional support. Mathematics at the secondary level is the subject with the highest number of lessons after Slovene language, it is complex, abstract and increasingly less related to material reality. At this level, it is no longer possible to strictly separate the overcoming of pupils' learning difficulties in individual mathematical topics from the teaching support, which is why it is essential to have the effective collaboration of different professionals who bring competencies and knowledge from their own areas of expertise to the learning process. Currently, individualised intervention as intensive and specific additional support in mathematics outside the classroom is the predominant approach in Slovenian elementary schools, but experts point out that in order to achieve optimal learning performance as well as social inclusion, more support should be provided within the classroom (Pulec Lah and Košir, 2015). Some Slovenian schools are starting this kind of practice, which requires a lot of teamwork between the subject mathematics teacher and the provider of intensive and specific additional support, as well as effective organisation. The aim of our research was to investigate in what form the selected schools implement the support, in what way, in what form and how often the special pedagogue and the subject mathematics teacher collaborate, in which cases, according to the professionals, individual or inclusive implementation is more appropriate, and what are the advantages and disadvantages of these two methods. According to the Regulations on the Qualifications of Teachers

and Professional Staff in the Primary School Education Programme (2022), different profiles of professional staff provide intensive and specific additional support, so our aim was to find out how the profile of the professional staff member providing each form of additional support is determined in the selected school. Through in-depth analysis of systematic observation of individual and inclusive mathematics teaching and interviews with subject mathematics teachers and special pedagogues, we found that effective collaboration between the subject mathematics teacher and the special pedagogue is key to achieving the learning goals in mathematics teaching. Quality delivery of mathematics teaching to low-achieving pupils requires quality teamwork between all those involved. In conclusion, we suggest that mathematics teachers and special pedagogues should work together to plan mathematics lessons in the case of inclusive implementation, especially for more challenging content. If the special pedagogue is providing individual support to the pupil, we suggest that in inclusive lessons, the special pedagogue should be present during the mathematics teacher's explanation and should keep a record, in the form prepared for this purpose, of key mathematical vocabulary, representations of concepts and procedures, problem chaining and other relevant guidance that can be used when working with the pupil outside the classroom in order to ensure consistency in the treatment of the concept. This is a prerequisite for the special educator to establish a dialogue with the pupils, which is further developed in accordance with the special education profession. When a pupil receives a disjointed professional and special didactic way of support, the results are significantly worse, despite the efforts of the educational staff, than when the mathematics teacher and the special educator work closely together in a pupil-centred way.

Keywords: special pedagogue, subject mathematics teacher, intensive and specific additional support, professional teamwork, inclusion

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Pomen sodelovanja učitelja matematike in specialnega pedagoga pri nudenju pomoči učencem z nižjimi dosežki pri matematiki

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Povzetek. Število učencev, ki imajo prilagojeno izvajanje pouka oz. so deležni dodatne strokovne pomoči pri matematiki vsako leto narašča. Zagotavljanje kakovostnega izvajanja pomoči za učence, ki imajo nižje dosežke pri matematiki, je na področju izobraževanja matematike vedno bolj aktualno, med drugim se pojavlja vprašanje, kako zagotoviti kompetentne strokovnjake za izvajanje dodatne strokovne pomoči. Matematika na predmetni stopnji obsega takoj za slovenščino največ ur, je zahtevna, abstraktna ter vedno manj vezana na materialno realnost. Na tej stopnji pri matematiki ni več mogoče strogo ločiti premagovanja primanjkljajev v znanju učencev pri posameznih matematičnih vsebinah od učne pomoči, zato je nujno učinkovito sodelovanje različnih strokovnjakov, ki v učni proces doprinesejo kompetence in znanja s svojega strokovnega področja. Trenutno v slovenskih osnovnih šolah prevladuje individualna izvedba dodatne strokovne pomoči pri matematiki izven razreda, vendar strokovnjaki poudarjajo, da bi bilo tako z vidika doseganja optimalne učne uspešnosti kot tudi socialne vključenosti potrebno pomoč učencu v večji meri nuditi znotraj razreda (Pulec Lah in Košir, 2015). Nekatere slovenske šole pričenjajo s tovrstno prakso, ki pa zahteva veliko timskega sodelovanja predmetnega učitelja matematike in izvajalca dodatne strokovne pomoči ter učinkovito organizacijo. Cilj naše raziskave je bil raziskati, v kakšni obliki na izbrani šoli izvajajo dodatno strokovno pomoč, na kakšen način, v kakšni obliki in kako pogosto pri tem sodelujeta specialni pedagog ter predmetni učitelj matematike, v katerih primerih je po mnenju izvajalcev bolj ustrezna individualna ali inkluzivna izvedba ter katere so njune prednosti in pomanjkljivosti. Skladno s Pravilnikom o izobrazbi učiteljev in strokovnih delavcev v izobraževalnem programu osnovne šole (2022) dodatno strokovno pomoč izvajajo različni profili strokovnih delavcev, zato

je bil naš namen ugotoviti, na kakšen način določajo profil strokovnega delavca za izvajanje posamezne oblike dodatne strokovne pomoči na izbrani šoli. S poglobljeno analizo sistematičnega opazovanja individualnega in inkluzivnega poučevanja matematike in intervjujev s predmetnimi učitelji matematike ter specialnimi pedagogi smo ugotovili, da je učinkovito sodelovanje med predmetnim učiteljem matematike in specialnim pedagogom ključno za doseganje učnih ciljev pri poučevanju matematike. Kakovostna izvedba poučevanja učencev z nižjimi dosežki pri matematiki zahteva kakovostno timsko sodelovanje vseh udeleženih. V zaključkih predlagamo, da bi učitelji matematike in specialni pedagogi v sodelovanju načrtovali učne ure za matematiko v primeru inkluzivne izvedbe, predvsem pri zahtevnejših vsebinah. V kolikor specialni pedagog nudi učencu individualno strokovno pomoč predlagamo, da je ob inkluzivnem izvajanju učnih ur specialni pedagog prisoten pri razlagi učitelja matematike in si sproti v, za ta namen pripravljen, obrazec vpisuje ključno matematično besedišče, reprezentacije pojmov in postopkov, veriženje nalog ter ostale pomembne napotke, ki jih lahko uporabi pri delu z učencem izven razreda z namenom, da zagotovi konsistentnost obravnavanja pojma. To specialnemu pedagogu predstavlja predpogoj za vzpostavitev dialoga z učenci, ki pa ga nadalje razvije skladno s specialno pedagoško stroko. Kadar je učenec deležen nepovezanega strokovnega in specialno didaktičnega načina nudenja pomoči, so rezultati kljub prizadevanjem pedagoških delavcev bistveno slabši, kot če učitelj matematike in specialni pedagog tesno sodelujeta osredinjeno na učenca.

Ključne besede: specialni pedagog, predmetni učitelj matematike, dodatna strokovna pomoč, sodelovanje strokovnjakov, inkluzija

Teacher's beliefs about teaching mathematics and teaching practice in Croatian primary schools

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Abstract. Nowadays, the importance of teaching mathematics through problem solving and problem tasks is emphasized on a very large scale. On the other hand, many practitioners indicate children's difficulties in problem solving.

This paper brings primary school teacher's beliefs about teaching mathematics and how they approach the realization of their teaching practice. Teacher's beliefs about teaching mathematics are categorized from traditional, primary traditional, mix of traditional and non-traditional, primary non-traditional to non-traditional model. Non-traditional models include problem-solving orientation to teaching practice and solving problem tasks in different stages of lessons. The goal of this research is to determine the correlation between teacher's beliefs and their approach to teaching practice. This research involved 184 teachers in primary schools in Croatia. The results have shown that primary school teacher's beliefs in Croatia about teaching mathematics converges to non-traditional models but their approach to teaching practice converges to traditional models.

Keywords: problem-solving teaching practice, problem tasks, traditional teaching practice, non-traditional teaching practice

Uvjerenja učitelja razredne nastave o poučavanju matematike i nastavna praksa u hrvatskim osnovnim školama

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Sažetak. U današnje vrijeme se u velikoj mjeri naglašava važnost problemskog pristupa nastavi matematike i korištenja problemskih zadataka. S druge strane mnoga iskustva praktičara ukazuju na dječje teškoće u rješavanju problemskih zadataka.

Ovaj rad donosi uvjerenja učitelja razredne nastave o pristupu poučavanju matematike i načine kojima pristupaju realizaciji nastave. Uvjerenja učitelja o poučavanju matematike i pristupi realizaciji nastave matematike kategoriziraju se od tradicionalnog, primarno tradicionalnog, mješovitog tradicionalnog i netradicionalnog, primarno netradicionalnog do netradicionalnog modela. Netradicionalni modeli uključuju problemsku orijentaciju nastave matematike i rješavanje problemskih zadataka u različitim etapama nastavnog sata. Cilj ovog istraživanja je utvrditi korelaciju između uvjerenja o poučavanju matematike i pristupa nastavnoj praksi. U istraživanju je sudjelovalo 184 učitelja razredne nastave iz osnovnih škola u Republici Hrvatskoj. Rezultati istraživanja pokazali su da su uvjerenja učitelja razredne nastave u RH o poučavanju matematike usmjerena prema netradicionalnim modelima dok je pristup realizaciji nastave većinom tradicionalan.

Ključne riječi: problemska nastava, problemski zadaci, tradicionalna nastava, netradicionalna nastava

The importance of thought transformations in learning geometry based on Van Hiele's theory

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Abstract. In mathematics lessons, geometry poses greater difficulties for students than other mathematical content, and one of the key reasons is the development and ability of spatial sense. Many empirical studies as well as theoretical statements, for example van Hiel's theory, indicate that the ability of thought transformations, as part of spatial ability, is a key predictor in the development of quality geometric reasoning and in general the perception of two-dimensional and three-dimensional objects and geometric transformations. During education, students' knowledge goes to higher levels such as analysis and informal deduction only through targeted activities. Spatial ability is not directly included in the mathematics curriculum of lower grades, neither in terms of content nor as a learning outcome, but it is necessary to include it in mathematics teaching activities. The basic predisposition to include spatial ability in teaching is the teacher's ability to perform geometric transformations. Topics such as the recognition of geometric figures and solids, the construction of perpendicular or parallel lines, types of triangles, etc., present a challenge in teaching if the visual representation of a geometric object is transformed in relation to the representation that the child first encounters in learning. In order for the teacher to be able to respond to the challenge, his spatial ability and geometric reasoning must correlate with beliefs about the purpose and importance of geometric content in teaching mathematics. The analysis presented in this paper was carried out on a sample of teacher study students who had geometry content as part of mathematics courses. The survey questionnaire examined their levels of geometric reasoning according to van Hiel's theory, geometric

transformation abilities and attitudes about geometry in mathematics classes in lower grades of primary school.

Keywords: geometry, Van Hiele's theory, spatial ability, thought transformations

Važnost misaonih transformacija u učenju geometrije na temelju Van Hieleove teorije

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Sažetak. U nastavi matematike geometrija učenicima predstavlja veće teškoće od ostalih matematičkih sadržaja, a jedan od ključnih razloga je razvoj i sposobnost prostornog zora. Mnoga empirijska istraživanja kao i teorijske postavke, primjerice van Hielova teorija, ukazuju da je sposobnost misaonih transformacija, kao dio prostornog zora, ključni prediktor u razvoju kvalitetnog geometrijskog mišljenja i općenito percepcije dvodimenzionalnih i trodimenzionalnih objekata, te geometrijskih transformacija. Tijekom obrazovanja učenikova znanja prelaze u više razine kao što su analiziranje i neformalna dedukcija samo ciljanim aktivnostima. Prostorni zor nije direktno uključen u kurikulum matematike nižih razreda, ni sadržajno niti kao ishod učenja, no nužno ga je uključivati u aktivnosti u nastavi matematike. Osnovna predispozicija uključivanja prostornog zora u nastavu je misaona sposobnost učitelja za izvođenje geometrijskih transformacija. Teme poput prepoznavanja geometrijskih likova i tijela, konstrukcije okomitih i paralelnih pravaca, vrste trokuta i sl. predstavljaju izazov u poučavanju ako je vizualni prikaz geometrijskog objekta transformiran u odnosu na prikaz kojim se dijete prvi put susreće u učenju. Da bi učitelj mogao odgovoriti izazovu njegov prostorni zor i geometrijsko mišljenje mora korelirati s uvjerenjima o svrhovitosti i važnosti geometrijskih sadržaja u nastavi matematike. Istraživanje prezentirano u ovom radu provedeno je na uzorku studenata učiteljskog studija koji su u okviru matematičkih kolegija imali geometrijske sadržaje. Anketnim upitnikom ispitale su se njihove razine geometrijskog mišljenja prema van Hieleovoj teoriji, sposobnosti izvođenja geometrijskih transformacija i stavovi o geometriji u nastavi matematike nižih razreda osnovne škole.

Ključne riječi: geometrija, van Hieleova teorija, prostorni zor, misaone transformacije

Analysis of logical operational abilities and logical reasoning among university students

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Abstract. Empirical research results show that the system of logical operation and conclusion schemes do not become complete even in adulthood. Understanding the school or university curriculum cannot be successful without the use of the most important inference chains. Proofs, reasoning, and refutation are necessary in the case of several subjects, as scientific language uses a large number of logical operations. Our 2022 end-of-year research measured general basic logical knowledge and operations in mathematical, physical, chemical, and biological contexts on a sample of 246 students. Measured with the Kuder-Richardson 21 formula, our test is reliable, and its results are consistent. The students completed the tasks which required the correct conclusion in the highest proportion (69,02 %-ban), followed by the interpretation of at most/at least (63,41%), and at least the negation (the negation of the least, of exists and of for all) (29,91%). We can say that the correct interpretation did not become dominant in the case of all logical operations for these students. The students performed the highest proportion of logical operations in a mathematical environment (60,27 %), followed by biology and physics (58,73%), and the least often in chemistry (38,71%). After conducting a correlation test, we found that there is no significant difference between the correct answers of students from different departments, of students in the bachelor or master's programmes or between genders.

Keywords: basic knowledge of mathematics, logical operational skills, logical reasoning, knowledge transfer

Alapvető logikai műveleti készségek és a logikai érvelés vizsgálata egyetemi hallgatók körében

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Absztrakt. Az empirikus kutatási eredmények azt mutatják, hogy a logikai műveleti és következtetési sémák, ennek rendszere a felnőttkorra sem válik teljessé. Az iskolai, illetve egyetemi tananyag megértése a legfontosabb következtetési láncok használata nélkül nem lehet sikeres, a bizonyítások, az érvelés, a cáfolás gondolatmenetei szükségesek több tantárgy esetén, a tudományos szaknyelv nagy számban alkalmaz logikai műveleteket. A 2022-es év végi kutatásunk egy 246 fős hallgatói mintán általános logikai alapismereteket és műveleteket mért matematikai, fizikai, kémiai, illetve biológiai szövegkörnyezetben. A Kuder-Richardson 21-es formulával mérve a tesztünk megbízható, az eredményei konzisztensek. A hallgatók a legnagyobb arányban a helyes következtetést igénylő feladatokat teljesítették (69,02 %-ban), ezt követi a legfeljebb/legalább értelmezése (63,41%), illetve legkevésbé a tagadást (a legalább, a létezik, a minden tagadását) (29,91%). Elmondható, hogy nem minden logikai művelet esetében vált a helyes értelmezés dominánssá ezeknél a hallgatóknál. A hallgatók a legnagyobb arányban a logikai műveleteket matematika környezetben teljesítették (60,27 %), ezt követi a biológia, fizika (58,73%), legkevésbé a kémiában tudják alkalmazni (38,71%). Összefüggésvizsgálatokat végezve azt kaptuk, hogy nincs szignifikáns különbség a különböző szakon tanuló hallgatók helyes válaszai között, az alap-, illetve a mesterképzésben lévő hallgatók teszten elért jó válaszainak száma között, illetve a nemek között.

Kulcsszavak: matematikai alapismeretek, logikai műveletek, logikai következtetés, tudástranszfer

The specifics of solving mathematical tasks using a model based on the Tablet-Human Hybrid Model of Avatar in university face-to-face teaching

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Abstract. Pairwork is a social form of teaching. Communication between paired students becomes a valuable exchange of information and active and effective work that contributes to better understanding and appropriation of course content. Cooperative learning improves mathematical achievements and attitudes toward mathematics.

The Tablet-Human Hybrid Model of Avatar (T-HHMA) has been tested in primary school. In the form of an avatar, the model enables classroom attendance and participation for absent students who are physically unable to attend due to illness, isolation, or other reasons. The model utilized a present classmate, who paired his tablet with the tablet of an absent student via an audio-video connection, as an agent. The agent's role was to fulfil the absent student's requests. The application of this model has never been tested in university classrooms.

The new model, which is still in the development phase, introduces pairwork by pairing the absent student with his or her agent, thus opening up the possibility of taking advantage of cooperative learning.

The specifics of solving mathematical tasks using the model are discussed. Data are collected through participant observation and interviews.

Keywords: avatar, cooperative learning, model, pairwork, university face-to-face classes

Specifičnosti rješavanja matematičkih zadataka korištenjem modela zasnovanog na hibridno humanom-tabletnom modelu avatara u fakultetskoj nastavi uživo

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Sažetak. Rad u paru jedan je od socijalnih oblika nastave. Komunikacija između uparenih studenata postaje važna razmjena informacija te aktivan i učinkovit oblik rada koji pridonosi boljem razumijevanju i usvajanju sadržaja kolegija. Suradničko učenje poboljšava matematička postignuća i stavove prema matematici.

Hibridni humano-tabletni model avatara (T-HHMA) testiran je u osnovnoj školi. U obliku avatara, model omogućuje pohađanje nastave i sudjelovanje odsutnih učenika koji zbog bolesti, izolacije ili drugih razloga nisu u mogućnosti prisustvovati nastavi. U modelu se kao agent koristi učenik iz razreda koji je audio-video vezom upario svoj tablet s tabletom odsutnog učenika. Uloga agenta bila je ispunjavanje zahtjeva odsutnog učenika. Primjena ovog modela nije do sada testirana u sveučilišnoj nastavi.

Novi model, koji je još u fazi razvoja, uparujući odsutnog studenta sa svojim agentom uvodi socijalni oblik rada u paru, čime se stvara mogućnost iskorištavanja prednosti suradničkog učenja.

Obrađene su specifičnosti rješavanja matematičkih zadataka pomoću ovog modela. Podatci su prikupljeni metodom promatranja sudionika i intervjuiranjima.

Ključne riječi: avatar, suradničko učenje, model, rad u paru, sveučilišna nastava uživo

On the use of visualization when performing basic arithmetic operations

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Abstract. Previous studies suggest that there is a link between students visuospatial and arithmetic abilities, but current diagnostic tools generally do not include tasks to detect this. Therefore, an Erasmus+ project in higher education called "Diagnostic Tool in Mathematics" (DiToM) started in January 2023. The main goal of the project is to provide standardized diagnostic tools to monitor the mathematical performance of pupils from kindergarten throughout primary and secondary school, focusing on the application of visuospatial skills to arithmetic problems.

This paper aims to provide insight into the visuospatial skills pupils use when solving arithmetic problems, and presents the results of a preliminary study conducted on a sample of 4th and 5th grade pupils. The tasks used combine visuospatial skills and basic numerical skills, and serve as a screening tool. However, it should be mentioned that these tasks do not indicate any type of learning disability (such as developmental dyscalculia, dyslexia, dysgraphia, etc.) in the screened pupils.

Keywords: visuospatial skills, numerical skills, arithmetic operations, diagnostic tool, primary school

O uporabi vizualizacije prilikom izvođenja osnovnih aritmetičkih operacija

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Sažetak. Prethodna istraživanja ukazuju da postoji veza između vizualno-prostornih i aritmetičkih sposobnosti učenika, ali trenutni dijagnostički alati uglavnom ne uključuju zadatke za otkrivanje toga. Stoga je u siječnju 2023. godine započeo Erasmus+ projekt u visokom obrazovanju pod nazivom "Diagnostic Tool in Mathematics" (DiToM). Glavni cilj tog projekta je osmisliti standardizirane dijagnostičke alate za kontinuirano praćenje matematičkih postignuća učenika od vrtića do osnovne i srednje škole, s fokusom na uporabu vizualno-prostornih vještina u rješavanju aritmetičkih problema.

Cilj ovog rada je dati uvid u vizualno-prostorne vještine koje učenici koriste pri rješavanju aritmetičkih zadataka, te prikazati rezultate preliminarnog istraživanja provedenog na uzorku učenika 4. i 5. razreda. Zadani zadaci objedinjuju vještine vizualizacije i numeričke vještine, te služe kao alat za provjeru. Međutim, treba spomenuti da zadaci ne ukazuju na bilo koju vrstu poteškoća u učenju (kao što su diskalkulija, disleksija, disgrafija, itd.) kod testiranih učenika.

Ključne riječi: vizualno-prostorne vještine, numeričke vještine, aritmetičke operacije, dijagnostički alat, osnovna škola

Decision trees in research on mathematics in elementary school

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Abstract. Mathematics is a compulsory subject and as such runs through the entire vertical of formal education, indicating its importance both in daily life and in its contribution to the accelerated development of modern society. Numeracy has not only played a great role in the past, but is also of great importance for the future, as it is necessary to understand the environment in which we live and is one of the foundations for the development of life skills of each individual. Educational data mining is a research area in which data mining methods are applied to educational data. Although the use of data mining in education has increased in recent years, the potential of these methods is not yet fully realized, especially in studies with primary school pupils. Decision trees, regression trees, and classification trees are methods that are applicable in this research area. The advantages of these methods are a very clear visual overview of the results for the description of the model, making them easy to analyze and interpret, and they can be very easily combined with other research methods. Since there is a lack of research that incorporates the aforementioned methods in mathematics research, especially in primary education, the purpose of this paper is to analyze the use of decision trees, regression trees, and classification trees in primary school mathematics research. A search of five academic databases (WoS, Scopus, Emerald, Eric and SpringerLink) found only 11 papers that meet the criteria. This review covers their analysis and answers research questions about the frequency of use of these methods and the purpose of their use.

Keywords: classification tree, decision tree, data mining, elementary school mathematics, regression tree

Stablo odlučivanja u istraživanju matematike u osnovnoj školi

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Sažetak. Matematika je obavezan nastavni predmet i kao takav proteže se tijekom cijele vertikale obrazovanja što ukazuje na njezinu važnost, kako u svakodnevnom životu, tako i u njenom doprinosu ubrzanom razvoju suvremenog društva. Osim što je imala veliku ulogu u prošlosti, matematička pismenost od iznimnog je značaja za budućnost jer je nužna za razumijevanje okruženja u kojem živimo i jedna je od osnova u razvoju životnih vještina svakog pojedinca. Rudarenje podataka u obrazovanju (engl. Educational data mining) je područje istraživanja u kojem se metode rudarenja podataka primjenjuju na podatke iz područja obrazovanja. Iako je uporaba rudarenja obrazovnih podataka posljednjih nekoliko godina u porastu, potencijal ovih metoda je još uvijek nedovoljno iskorišten, naročito u istraživanjima u koja su uključeni učenici osnovne škole. Stabla odlučivanja, regresijska stabla i klasifikacijska stabla su metode primjenjive u ovom području istraživanja. Prednosti uporabe ovih metoda je vrlo jasan vizualni pregled rezultata za opis modela, što ih čini jednostavnima za analizu i interpretaciju, a vrlo lako se mogu kombinirati i s drugim metodama istraživanja. Kako je primijećen nedostatak istraživanja koji uključuju navedene metode u istraživanjima matematike, posebice u osnovnoškolskom obrazovanju, svrha ovog rada je analizirati upotrebu stabla odlučivanja, regresijskih i klasifikacijskih stabala u istraživanjima matematike u osnovnoj školi. Pretragom pet akademskih baza (WoS, Scopus, Emerald, Eric i SpringerLink) pronađeno je svega 11 radova koji zadovoljavaju zadane kriterije. Ovaj pregled literature obuhvaća njihovu analizu te daje odgovor na istraživačka pitanja o učestalosti korištenja ovih metoda i svrsi njihove uporabe.

Ključne riječi: klasifikacijska stabla, stablo odlučivanja, osnovnoškolska matematika, regresijska stabla

Hybrid teaching and outcome indicators of the state graduation exam in mathematics

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Abstract. Although the use of multimedia in terms of modernization in teaching process has been for some time a subject of study in the fields of pedagogy and didactics, the educational system in the Republic of Croatia had to engage in a specific way in the field of hybrid learning strategies in the past three years. Two devastating earthquakes and several waves of the advanced pandemic of the COVID-19 virus caused the need for alternative teaching models as opposed to classical and contact ones. In addition to the above, it required adaptation and realization of new competencies for both students and teachers, and even though it ensured uninterrupted education, it also pointed to certain problems and negative consequences. This paper provides an overview of the requirements, challenges, advantages and disadvantages of hybrid teaching, specifically showing the results of the state graduation exam in mathematics among students of Croatian schools, who took the exam after attending school in the mentioned conditions of the specific social situation. Recommendations for science and practice based on the indicated comparisons suggest the observation of the mentioned causes and related phenomena in the outcomes and success of teaching both in this and other school subjects and at all levels of upbringing and education.

Keywords: hybrid teaching, hybrid teaching of mathematics, state graduation exam, teaching during the pandemic

Hibridno izvođenje nastave i pokazatelji ishoda državne mature iz matematike

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Sažetak. Iako je uporaba multimedija u svrhu osuvremenjivanja nastave već određeno vrijeme predmetom proučavanja na poljima pedagogije i didaktike, odgojno-obrazovni sustav u Republici Hrvatskoj se u protekle tri godine na specifičan način morao angažirati na polju strategija hibridnog učenja. Dva razorna potresa i nekoliko valova uznapredovale pandemije virusa COVID-19 uzrokovali su potrebu alternativnih modela nastave nasuprot klasičnim i kontaktnim. Osim što je navedeno zahtijevalo potrebe prilagodbe i ostvarivanja novih kompetencija kako učenika tako i nastavnika i iako je osiguralo neprekidanje tijeka školovanja, ukazalo je i na određene probleme i negativne posljedice. Ovaj rad donosi prikaz zahtjeva, izazova, prednosti i nedostataka hibridne nastave, konkretno prikazujući rezultate ishoda državne mature iz matematike među učenicima hrvatskih škola, a kojima je neposredno pisanju ispita prethodila nastava u navedenim uvjetima specifične društveno-socijalne situacije. Preporuke za znanost i praksu na temelju ukazanih usporedbi sugeriraju i promatranje navedenih uzroka i povezanih pojava u ishodima i uspješnosti nastave kako u ovom, tako i drugim nastavnim predmetima na svim razinama odgoja i obrazovanja.

Ključne riječi: hibridna nastava, hibridna nastava matematike, državna matura, nastava u vrijeme pandemije

Requirements in digital mathematics textbooks: Development of the framework for task analysis

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Abstract. Textbook tasks create student opportunities for learning mathematics, because they may limit or broaden their views and may influence the way students think through their engagement. Since textbook tasks are widely used by students in mathematics education, it is important to examine their features and requirements. While the tasks in printed mathematics textbooks have been extensively examined worldwide, the requirements of the tasks in digital textbooks are yet to be analyzed and systematically developed. Digital educational materials provide new opportunities for teaching and learning, such as dynamics, personalization, cooperation, interactivity, formative assessment, etc. This study presents the multi-dimensional framework for analyzing tasks from digital mathematics textbooks. The main categories of the framework are developed on the basis of the literature review and the further e-textbook analyses brought refining of its subcategories. The study also encompasses the analysis of tasks in geometry chapters of digital textbooks in primary grades in Croatia. The results show that the examined e-tasks do not provide a full range of features according to the framework applied. Personalization, cooperation, and exploring requirements are not used in the textbooks examined. Still, some e-textbooks contained dynamics within tasks containing drawing or dragging points in the plane. Regarding the task form, the textbooks contained multiple-choice, fill-in-the-blank, matching, and true/false tasks. These results imply that the new opportunities afforded by digital tasks are not fully realized and the findings reveal the potential of digital tasks as a new area to be further explored and developed.

Keywords: digital textbooks, textbook tasks, task analysis, geometry

Zahtjevi u digitalnim matematičkim udžbenicima: Razvoj okvira za analizu zadataka

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Sažetak. Udžbenički zadatci učenicima pružaju prilike za učenje matematike jer zadatci mogu ograničiti ili proširiti njihove poglede te svojim angažmanom utjecati na način na koji učenici razmišljaju. Budući da učenici u nastavi matematike često koriste zadatke iz udžbenika, važno je ispitati njihove karakteristike i zahtjeve. Dok su zadatci u tiskanim udžbenicima matematike opsežno istraženi diljem svijeta, zahtjeve zadataka u digitalnim udžbenicima tek treba analizirati i sustavno razvijati. Digitalni obrazovni materijali pružaju nove mogućnosti za poučavanje i učenje, kao što su dinamika, personalizacija, suradnja, interaktivnost, formativno vrednovanje i sl. U ovom radu se prikazuje višedimenzionalni okvir za analizu zadataka iz digitalnih udžbenika matematike. Glavne kategorije okvira razvijene su na temelju pregleda literature, a daljnje analize digitalnih udžbenika donijele su doradu njegovih potkategorija. Rad također obuhvaća analizu zadataka u poglavljima geometrije digitalnih udžbenika u razredima primarnog obrazovanja u Republici Hrvatskoj. Rezultati pokazuju da ispitani digitalni zadatci ne pružaju potpuni spektar mogućnosti prema primijenjenom okviru. Zahtjevi personalizacije, suradnje i istraživanja ne koriste se u ispitanim udžbenicima. Ipak, neki digitalni udžbenici sadrže dinamiku unutar zadataka koji su sadržavali crtanje ili povlačenje točaka u ravnini. Što se tiče tipa zadataka, ispitani udžbenici su sadržavali zadatke s višestrukim izborom, popunjavanjem, spajanjem te zadatke tipa točno/netočno. Ovi rezultati upućuju na to da nove mogućnosti koje pružaju digitalni zadaci nisu u potpunosti ostvarene te ukazuju na potencijal digitalnih zadataka kao novog područja koje treba dalje istraživati i razvijati.

Ključne riječi: digitalni udžbenici, zadaci iz udžbenika, analiza zadataka, geometrija

Word problems in the textbooks for primary mathematics education in Croatia

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Abstract. Word problems are an indispensable part of mathematics education. In the literature, they are tasks set up in real-life, imaginary or mathematical contexts solved using arithmetic operations on numerical data. Word problems can be classified using different criteria: the type of context, arithmetic and semantic structures, and the function of graphics accompanying the task. We performed a content analysis of selected textbook units covering all arithmetic operations across six textbook editions and all four grades of primary education. The results are hence informative about the implemented methodology and indicative of the distribution of different types of tasks. The classification proposed in this paper is exhaustive, unambiguous and appropriate for the analysis of word problems in primary mathematics textbooks regardless of grades and editions. Results of the study suggest that particular types of contextual problems are more common than others and that problems that have an arithmetic structure with operand unknown or multi-step problems are more commonly mathematical than contextual. Providing diverse opportunities in working with contextual problems is relevant knowledge for teachers, textbook authors and policymakers hence further research following the proposed methodology should be undertaken.

Keywords: word problem, contextual problem, graphic representation, primary mathematics education, mathematics textbook, content analysis

Tekstualni zadaci u udžbenicima za razrednu nastavu matematike u Republici Hrvatskoj

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Sažetak. Tekstualni zadaci su neizostavan dio matematičkog obrazovanja. U literaturi su to zadaci postavljeni u stvarnom, imaginarnom ili matematičkom kontekstu koji se rješavaju pomoću aritmetičkih operacija na numeričkim podacima. Tekstualni zadaci se mogu klasificirati prema različitim kriterijima: vrsti konteksta, aritmetičkoj i semantičkoj strukturi te funkciji grafike koja prati zadatak. Proveli smo analizu sadržaja odabranih udžbeničkih jedinica pokrivajući sve računske operacije u šest udžbeničkih izdanja i sva četiri razreda osnovnog obrazovanja. Rezultati su zato informativni o primijenjenoj metodologiji i indikativni o raspodjeli različitih tipova zadataka. Klasifikacija predložena u ovom radu iscrpna je, nedvosmislena i prikladna za analizu tekstualnih zadataka u udžbenicima matematike za niže razrede osnovne škole bez obzira na izdanje i razred. Rezultati studije sugeriraju da su određeni tipovi kontekstualnih zadataka češći od drugih i da su problemi koji imaju aritmetičku strukturu s nepoznatim operandom ili problemi s više koraka češće matematički nego kontekstualni. Pružanje različitih prilika u radu s kontekstualnim problemima relevantno je znanje za nastavnike, autore udžbenika i kreatore politika, stoga bi trebalo poduzeti daljnja istraživanja slijedeći predloženu metodologiju.

Ključne riječi: tekstualni zadatak, kontekstualni problem, grafički prikaz, osnovnoškolsko matematičko obrazovanje, matematički udžbenik, analiza sadržaja

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Digital pedagogy and teaching mathematics – Trends, perspectives, limitations and challenges

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Abstract. The increased use of modern digital technologies in the teaching process in recent years has initiated, among other things, the accelerated development of new areas of pedagogy. In education research, the field of digital pedagogy is increasingly represented, and it is shown that there is a significant potential for its application in the teaching of mathematics, which is also supported by numerous examples of recent practice of teaching and learning mathematics. The paper clarifies the concept of digital pedagogy and its potential in solving the traditional challenges of teaching mathematics, as well as the possibility of developing new strategies and approaches to the design and development of an innovative learning process.

Various possibilities of using digital technologies that teachers can use in efforts to improve the quality of the teaching process are suggested, and the shortcomings and some new challenges brought by the increased use of ICT in mathematics teaching are highlighted. The application of digital educational technologies in the teaching of mathematics and the changed role of the mathematics teacher are critically reviewed, as well as the knowledge and skills teacher needs in order for students to learn mathematics more efficiently and develop optimally through the various forms of formal, non-formal and informal learning.

Keywords: digital pedagogy, teaching mathematics, teacher competencies

Digitalna pedagogija i nastava matematike – Trendovi, perspektive, ograničenja i izazovi

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Sažetak. Povećana upotreba suvremenih digitalnih tehnologija u nastavnom procesu posljednjih godina inicirala je, među ostalim i ubrzani razvoj novih područja pedagogije. U istraživanjima odgoja i obrazovanja sve je više zastupljeno područje digitalne pedagogije te se pokazuje kako postoji značajan potencijal primjene u nastavi matematike, što i podupiru brojni primjeri recentne prakse učenja i poučavanja matematike. U radu se pojašnjava koncept digitalne pedagogije i njezin potencijal u rješavanju tradicionalnih izazova nastave matematike te mogućnosti razvijanja novih strategija i pristupa oblikovanju i razvoju inovativnog procesa učenja.

Sugeriraju se različite mogućnosti korištenja digitalnih tehnologija koje nastavnici mogu koristiti u nastojanjima unapređenja kvalitete nastavnog procesa te se ističu nedostaci i neki novi izazovi koje sa sobom donosi povećana uporaba IKT-a u nastavi matematike. Kritički se preispituje primjena digitalnih obrazovnih tehnologija u nastavi matematike te promijenjena uloga nastavnika matematike, kao i znanja i vještine koje su mu potrebne kako bi učenici efikasnije učili matematiku te se optimalno razvijali kroz različite formalne, neformalne i informalne oblike učenja.

Ključne riječi: digitalna pedagogija, nastava matematike, kompetencije nastavnika

Sudoku puzzle

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Abstract. Sudoku is a popular game consisting of 9x9 squares divided into 9 smaller squares. The goal of the game is to fill the empty squares with numbers from 1 to 9 so that each number is unique in each row, column and smaller 3x3 square.

The origins of Sudoku can be traced back to the late 19th century, when a Swiss mathematician Leonhard Euler developed a puzzle called the "Latin square". In the 1970s, a puzzle designer named Howard Garns created a modern version of the Latin square puzzle, which he called "Number Place". In the 1980s, Number Place was introduced to Japan by the publisher Nikoli, who gave it the name "Sudoku", which means "single number" or "number alone". The puzzle became incredibly popular in Japan, and in the late 1990s, it was exported to other countries and became a global phenomenon. Today, Sudoku is one of the most popular puzzle games in the world, with millions of people enjoying it daily in newspapers, books, and online.

Logical puzzles, including Sudoku puzzles, can have a positive impact on child development. Ways in which solving these types of puzzles can benefit children are: cognitive development, memory improvement, patience and persistence and stress reduction. Overall, solving logical puzzles can be a fun and engaging way for children to develop important cognitive and life skills.

There are several techniques for solving Sudoku, like Forced cells, Twins, X-wings, Ariadne's thread or Forcing chain.

Given a completed Sudoku square, transformations that we can apply without disrupting its Sudokuness are: relabeling the digits, permuting the rows in a band or the columns in a pillar, permuting the blocks in a given band or pillar or any rotation or reflection.

Any transformation of Sudoku square that preserves Sudokuness can be expressed as a combination of the items on the list. That two Sudoku squares are

fundamentally different means that there is no combination of items on the list allowing us to transform one into the other. Russell and Jarvis found that the total number of fundamentally different Sudoku squares is 5,472,730,538. The different puzzles, with different difficulties, can have the same solution square. If two puzzles are related by transformation from the list, then those two puzzles should be exactly the same level of difficulty and require exactly the same sort of solving techniques.

Providing more than 17 clues may make the puzzle easier to solve, but providing fewer than 17 clues makes it impossible to guarantee that the puzzle has a unique solution. Therefore, 17 clues are the minimum number of clues required to ensure that a Sudoku puzzle can be solved, although there is no mathematical proof for that.

The Sudoku puzzle could be also recast as a problem in graph coloring. Think of each of the 81 cells in the grid as a vertex. Each of these vertices generates a zone consisting of the 8 other vertices in its row, the 8 other vertices in its column, and the 4 other vertex in its block that is not in the same row or column. For each vertex, add connecting edges to every other vertex in its zone. The Sudoku graph has 81 of vertex and 810 edges. Cells appearing in the same Sudoku zone must contain different digits. We shall represent this fact in the graph by assigning colors to the vertices, with connected vertices receiving different colors. The chromatic number of the Sudoku graph is 9.

Nowadays, there are many different types of Sudoku puzzles, each with its own unique rules and variations. Here are some examples: Classic Sudoku, Shidoku, Mini Sudoku, Samurai Sudoku, Diagonal or X Sudoku, Killer Sudoku, Irregular Sudoku, Windoku, Hyper Sudoku, Thermometer Sudoku, Knight Sudoku, Consecutive Sudoku, Toroidal Sudoku, Even-Odd Sudoku, Sandwich Sudoku, Skyscraper Sudoku and many others combination of different type of Sudokus.

Keywords: Sudoku puzzle, Sudoku variants, techniques for solving Sudoku, graph coloring, fundamentally different Sudoku squares

Sudoku

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Sažetak. Sudoku je popularna igra koja se sastoji od kvadrata veličine 9x9 polja podijeljenog na 9 manjih kvadrata. Cilj igre je popuniti prazna polja brojevima od 1 do 9 tako da svaki broj bude jedinstven u svakom retku, stupcu i manjem kvadratu 3x3.

Počeci Sudokua sežu u kasno 19. stoljeće, kada je švicarski matematičar Leonhard Euler razvio zagonetku nazvanu "Latinski kvadrat". U 1970-ima, dizajner slagalice po imenu Howard Garns stvorio je modernu verziju slagalice s latinskim kvadratom, koju je nazvao "Number Place". U 1980-ima je izdavač Nikoli uveo slagalicu u Japan, te je nazvao "Sudoku", što znači "jedan broj" ili "samo broj". Zagonetka je postala nevjerojatno popularna u Japanu, a kasnih 1990-ih izvezena je u druge zemlje i postala globalni fenomen. Danas je Sudoku jedna od najpopularnijih zagonetka na svijetu, s milijunima ljudi koji svakodnevno uživaju u njoj u novinama, knjigama i na internetu.

Logičke zagonetke, uključujući Sudoku, mogu pozitivno utjecati na razvoj djeteta. Načini na koje rješavanje ovih vrsta zagonetki može koristiti djeci su: kognitivni razvoj, poboljšanje pamćenja, strpljenje i upornost te smanjenje stresa. Općenito, rješavanje logičkih zagonetki može biti zabavan i privlačan način da djeca razviju važne kognitivne i životne vještine.

Postoji nekoliko tehnika za rješavanje Sudokua, poput prisilnih ćelija, blizanaca, X-krila, Arijadnine niti ili prisilnog lanca.

S obzirom na popunjeni Sudoku kvadrat, transformacije koje možemo primijeniti bez narušavanja njegove esencije su: preimenovanje znamenaka, permutacija redaka u traci ili stupaca u stupu, permutacija blokova u danoj traci ili stupu ili bilo kakva rotacija ili zrcaljenje.

Svaka transformacija Sudoku kvadrata koja čuva esenciju može se izraziti kao kombinacija navedenih transformacija. Kada kažemo da su dva Sudoku kvadrata fundamentalno različita, to znači da ne postoji kombinacija transformacija koja nam omogućuje prijelaz iz jednog Sudoku kvadrata u drugi. Russell i Jarvis otkrili su da je ukupan broj fundamentalno različitih Sudoku kvadrata jednak 5.472.730.538. Različiti Sudokui, s različitim razinama težine, mogu imati isti kvadrat kao rješenje. Ako su dva Sudoku kvadrata povezana transformacijama koje čuvaju esenciju problema, tada bi te dvije zagonetke trebale biti potpuno iste razine težine i zahtijevati potpuno istu vrstu tehnika rješavanja.

Zadavanje više od 17 tragova na početku Sudokua može olakšati rješavanje zagonetke, ali zadavanje manje od 17 tragova onemogućuje jamstvo da zagonetka ima jedinstveno rješenje. Stoga je 17 tragova minimalan broj tragova koji su potrebni da bi se osiguralo da se Sudoku može riješiti, iako za to nema matematičkog dokaza.

Rješavanje Sudoku zagonetke može se prikazati i kao problem u bojanju grafova. Svako od 81 polja u rešetki predstavimo kao jedan od vrhova grafa. Svaki od ovih vrhova stvara zonu koja se sastoji od 8 drugih vrhova u istom retku, 8 drugih vrhova u istom stupcu i 4 druga vrha u istom bloku koji nisu u istom retku ili stupcu kao promatrani vrh. Za svaki vrh definiramo bridove koji ga povezuju sa svakim drugim vrhom u njegovoj zoni. Sudoku graf ima 81 vrh i 810 bridova. Polja koja se pojavljuju u istoj Sudoku zoni moraju sadržavati različite znamenke. Tu ćemo činjenicu predstaviti u grafu dodjeljivanjem boja vrhovima, pri čemu će povezani vrhovi dobiti različite boje. Kromatski broj Sudoku grafa je 9.

Danas postoji mnogo različitih inačica Sudoku zagonetki, svaka sa svojim jedinstvenim pravilima i varijacijama. Evo nekoliko primjera: Klasični Sudoku, Shidoku, Mini Sudoku, Samurai Sudoku, Diagonal ili X Sudoku, Killer Sudoku, Irregular Sudoku, Windoku, Hyper Sudoku, Thermometer Sudoku, Knight Sudoku, Consecutive Sudoku, Toroidal Sudoku, Even-Odd Sudoku, Sandwich Sudoku, Skyscraper Sudoku i mnoge druge kombinacije spomenutih vrsta Sudokua.

Ključne riječi: Sudoku slagalica, Sudoku inačice, tehnike rješavanja Sudokua, bojanje grafova, fundamentalno različiti Sudoku kvadrati

On the experience of a problem-posing activity with second grade primary school pupils

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Abstract. Representing quantities by numbers and describing the relationships between them using mathematical operations and relationships, in short, developing quantitative reasoning skills, is one of primary school mathematics education priorities. As part of a broader research project based on problem-oriented curriculum development, we are now investigating how a problem-oriented approach contributes to the development of quantitative reasoning. Our research focuses on authentic text-based problem-solving and problem-posing. In this presentation, we analyze the classroom activities of 2nd-grade primary school pupils to answer two questions: Is problem posing as a teaching method in mathematics classrooms applicable at the age under study? Does the problem-posing activity fulfill its role of providing information about the level of understanding of the mathematical concepts being taught? To answer these questions and draw pedagogical conclusions, we analyze the video recordings of the lesson planned by the research team.

Keywords: problem-posing, quantitative reasoning, problem-oriented teaching approach, primary school students, multiplication and division problems

Tapasztalatok a második évfolyamos tanulók problémalakotási tevékenységéről

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Absztrakt. A mennyiségek számokkal történő reprezentálása, valamint a mennyiségek közötti kapcsolatok leírása matematikai műveletekkel és relációkkal, röviden a mennyiségi érvelés képességének fejlesztése, az általános iskola alsó tagozatának kiemelt feladata. Egy problémaközpontú tananyagfeldolgozással foglalkozó átfogó kutatási projekt részeként most azt vizsgáljuk, hogy a problémaközpontú megközelítés hogyan járulhat hozzá a mennyiségi érvelés fejlődéséhez. Kutatásunk az autentikus szöveges feladatokon alapuló problémamegoldásra illetve a problémaalkotásra irányul. Ebben az előadásban második évfolyamos alsó tagozatos tanulók osztálytermi tevékenységét vizsgálva két kérdésre keressük a választ: Alkalmazható-e a problémaalkotás mint tanítási módszer a vizsgált korosztályban? Betölti-e a problémaalkotás azt a szerepét, hogy információt nyújt a tanított matematikai fogalmak megértésének aktuális szintjéről? A kérdések megválaszolásához és a pedagógiai következtetések megfogalmazásához a kutatócsoport által tervezett tanóráról készült videófelvételeket elemezzük.

Kulcsszavak: problémaalkotás, mennyiségi érvelés, problémaközpontú tananyagfeldolgozás, alsó tagozatos tanulók, szorzási és osztási problémák

Learner-generated drawings in mathematics: Who? When? How?

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Abstract. Learning as a generative activity involves making sense of the information to be learned through mental reorganization and integration with prior knowledge, thereby enabling individuals to apply acquired knowledge in new situations. It is promoted in recent times through different learning strategies. One of the strategies is the drawing strategy, which includes the skill of construction, which is also developed in mathematics lessons.

This paper presents the results of the initial knowledge assessment conducted at the Faculty of Civil Engineering of the University of Zagreb, in October 2022 with the aim of gaining insight into the development of construction skills as a mathematical activity. In this paper, the term "construction" refers to a mathematical activity that is performed in the mind with ideal objects and includes the activity of finding the sequence of construction steps while respecting the properties of various geometric concepts. The realization of the mathematical idea itself is carried out through drawing, which in this research is carried out with the permitted aids of a compass and a ruler (although in general, in mathematics lessons, drawing can be carried out with the help of other aids). The paper analyzes the results of the constructions of 111 first-year undergraduate students. Mathematical development of construction skills is of particular importance for students of technical fields because spatial problem situations from the profession are solved by applying constructive procedures through deepening and expanding existing geometric knowledge and concepts about space from the lower and secondary school. Along with the discussion of the test results that indicate that there are significant deficiencies in the adoption of fundamental concepts from plane geometry at the secondary level of education in Croatia, we make proposals for the selection and

design of construction tasks in the field of geometry of space that promote student learning and mathematical modelling through the use of precise mathematical explanations for the constructive procedures that are carried out. It is important to emphasize that the ability to perform geometric constructions accurately and independently, even in the age of increasing digitization, is particularly important for the development of spatial reasoning and should not be underestimated.

Keywords: geometric reasoning, drawing, graphical representation, mathematic modelling, visualization

Crteži rukom u matematici: Tko? Kada? Kako?

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Sažetak. Učenje kao generativna aktivnost uključuje davanje smisla informacijama koje se trebaju naučiti mentalnom reorganizacijom i integracijom s prethodnim znanjem, čime se pojedincima omogućuje primjena stečenog znanja u novim situacijama. Promovira se u novije vrijeme kroz različite strategije učenja. Jedna od strategija je strategija crtanja koja uključuje vještinu konstruiranja koja se razvija i u nastavi matematike.

U ovom radu predstavljeni su rezultati inicijalne provjere znanja provedene na Građevinskom fakultetu Sveučilišta u Zagrebu u listopadu 2022. godine s ciljem stjecanja uvida u razvijenost vještine konstruiranja kao matematičke aktivnosti. Pod pojmom konstrukcija u ovom radu podrazumijeva se matematička aktivnost koja se izvodi u umu s idealnim objektima i obuhvaća aktivnost pronalaženja slijeda koraka konstrukcije uvažavajući svojstva različitih geometrijskih koncepata. Sama realizacija matematičke ideje provodi se kroz crtanje koje se u ovom istraživanju provodi uz dopuštena pomagala šestar i ravnalo (iako se općenito se u nastavi matematike crtanje može izvoditi i uz pomoć drugih pomagala). U radu se analiziraju rezultati konstrukcija 111 studenata prve godine preddiplomskog studija. Matematičko razvijanje vještine konstruiranja od posebnog je značaja za studente tehničkih područja jer se prostorne problemske situacije iz struke rješavaju primjenom konstruktivnih postupaka kroz produbljivanje i proširivanje postojećih geometrijskih znanja i pojmova o prostoru iz niže i srednje škole. Uz raspravu o rezultatima provjere koji upućuju na to da postoje izrazite manjkavosti u usvojenosti temeljnih koncepata iz geometrije ravnine na razini visokog obrazovanja u Hrvatskoj, donosimo prijedloge za izbor i dizajn konstrukcijskih zadataka iz područja geometrije prostora koji promiču studentsko učenje i matematičko modeliranje kroz upotrebu preciznih matematičkih obrazloženja za konstruktivne postupke koji se

provode. Važno je naglasiti da je sposobnost preciznog i samostalnog izvođenja geometrijskih konstrukcija, čak i u doba sve veće digitalizacije, posebno važna za razvoj prostornog rasuđivanja i ne treba ju podcjenjivati.

Ključne riječi: geometrijsko rasuđivanje, crtanje, grafička reprezentacija, matematičko modeliranje, vizualizacija

Collective classroom climate in geometry lessons

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Abstract. The classroom is a social context in which students act and learn on a daily basis. Over time each classroom develops a specific social climate with different characteristics. Since the classroom climate refers to specific curricular and pedagogical components, it may differ for different school subjects but also for different topics (e.g., within mathematics). The focus of this paper is placed on examining the collective classroom social climate presented in students' drawings of their geometry lessons. The classroom social climate was analysed on the basis of three categories, namely Interpersonal relationship, Personal growth, and Order, in six elementary classes of the same school (two classes per Grades 3–5). The results showed some similarities and differences among participating classes. The differences among classes were reflected in teachers' enactment (teacher makes mathematics statements or asks mathematical questions), students' enactment (student discusses or listens passively), and teaching materials and tools specific for geometry lessons (e.g., 2D-shapes and models, 3D-solids and models, geometric tools). Furthermore, the results showed that the majority of classes, regardless of the grade level, presented the geometry lessons as being taught frontally with the teacher standing in front of the blackboard while teaching, and with students at their tables organized in the traditional classroom seating arrangement. The goal of the lesson was clearly shown in almost all drawings. In a wider sense, the similarities in the classroom climate both among different classes as well as grade levels may also imply some features of the school climate regarding geometry lessons which are discussed in the end.

Keywords: social climate, collective climate, geometry, elementary school, primary education

Kolektivna razredna klima u nastavi geometrije

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Sažetak. Učionica predstavlja društveni kontekst u kojem učenici svakodnevno djeluju i uče. Tijekom vremena svaki razred razvija specifičnu društvenu klimu s različitim karakteristikama. Budući da se razredna klima odnosi na specifične kurikularne i pedagoške komponente, ona se može razlikovati u različitim školskim predmetima, ali i tijekom učenja različitih tema (npr. unutar matematike). Fokus ovog rada stavljen je na ispitivanje kolektivne razredne klime koja je prikazana na učeničkim crtežima o satovima geometrije. Razredna socijalna klima analizirana je na temelju triju kategorija: Međuljudski odnosi, Osobni rast i Osobni razvoj, i to u šest razreda iste osnovne škole (po dva razreda od 3. do 5. razreda). Rezultati su pokazali neke sličnosti i razlike među ispitanim razredima. Razlike su se odnosile na pristup nastavnika (nastavnik daje matematičke izjave ili postavlja matematička pitanja), sudjelovanje učenika (učenik sudjeluje u diskusiji ili pasivno sluša) te na nastavne materijale i alate specifične za nastavu geometrije (npr. 2D oblici i modeli, 3D tijela i modeli, geometrijski pribor). Nadalje, rezultati su pokazali da je većina razreda, bez obzira na generaciju, prikazala nastavu geometrije u obliku frontalne nastave s nastavnikom koji stoji pred pločom dok predaje, a učenici sjede za svojim stolovima organizirani u tradicionalnom rasporedu sjedenja u učionici. Cilj nastavne lekcije bio je jasno prikazan na gotovo svim crtežima. U širem smislu, sličnosti u razrednoj klimi između različitih razreda, kao i među različitim generacijama, mogu podrazumijevati i neke značajke školske klime vezano uz nastavu geometrije, o čemu se diskutira na kraju rada.

Ključne riječi: društvena klima, kolektivna klima, geometrija, osnovna škola, osnovno obrazovanje

Mathematical teaching of preschool children through elements of performing arts

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Abstract. This paper intends to connect mathematics and elements of performing arts (dance, movement, play, music, etc.) in order to bring closer and improve the mathematical content of the preschool curriculum. It is known that the awareness of the importance of authentic teaching of mathematics in early preschool age is not sufficiently developed. One of the possible reasons is that educators are generally not methodologically sufficiently prepared for early mathematics, which implies the necessity of changes in the curriculum, that is, in the very approach and way of working. The emphasis is on developmentally appropriate teaching and the importance of initial understanding and acquisition of mathematical logic and concepts. The goal is to develop concrete mathematical examples using scenic elements in order to improve the existing curriculum. Mathematical - scenic procedures will be applied to the following examples: learning and comparing numbers, sets and relations between sets, and basic geometric content.

Keywords: mathematics, performing arts, preschool age, learning

Matematičko poučavanje djece predškolske dobi kroz elemente scenske umjetnosti

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Sažetak. Cilj rada bio je povezati matematiku i elemente scenske umjetnosti (ples, pokret, igra, glazba i sl.) kako bismo djeci rane i predškolske dobi približili određeni matematički sadržaj i učinili ga zabavnim, olakšavajući im učenje matematičkih sadržaja. Iz dosadašnje prakse, razvidno je kako djeca rane i predškolske dobi nisu u dovoljnoj mjeri upoznata s osnovnim matematičkim sadržajima. Jedan od mogućih razloga je što odgajatelji općenito nisu metodološki dovoljno pripremljeni za ranu matematiku, što implicira nužnost promjena u kurikulumu odnosno u samom pristupu i načinu rada s djecom. Stoga naglasak je stavljen na kronološki prikladnom poučavanju kao i važnosti inicijalnog razumijevanja i usvajanja matematičke logike i pojmova. U radu su razrađeni konkretni matematički primjeri pomoću scenskih elementa kako bi se unaprijedio postojeći kurikulum. Matematičko – scenski postupci primjenjivat će se na primjerima učenja i uspoređivanja brojeva, skupova i odnosa skupova i osnovnog geometrijskog sadržaja.

Ključne riječi: matematika, scenska umjetnost, predškolska dob, učenje

The role of mathematical picture books in teaching the concept of zero to first grade

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Abstract. Over the last two decades, mathematical picture books have increasingly emerged as an innovative teaching approach to support students' understanding of mathematical concepts. The baseline of this paper is a mathematical picture book entitled "70 cherries", which addresses first graders' understanding of the concept of zero. In the theoretical part, the role of this didactic tool for teaching mathematical concepts is presented, and next we focus on the content of our mathematical picture book, which, through an illustrative and imaginative story, introduces the reader to different aspects of the number zero: as a number at the beginning of a positive number line; as a digit in place value notation; as a neutral element in subtraction and addition and as a symbol to denote the absence of a quantity.

In the empirical part, we explore the different functions of the mathematical picture book in teaching the above concept: educational (from the perspective of upbringing and the perspective of learning) and motivational. The research aimed to find out how first-grade students recognise the role of the number zero in different contexts, how they accept it as a didactic tool and what the first-grade teachers think about the usefulness of the chosen mathematical picture book. The results of the study showed that the use of a mathematical picture book improves pupils' understanding of different aspects of the number zero. Pupils feel comfortable with the mathematical picture book and also recognise its educational aspect. The study also found that teachers recognise the "70 cherries" mathematics picture book as a useful didactic tool for teaching mathematics as well as other subjects. They recognise the motivational and educational function of this didactic tool, but

difficulties are found in the recognition of the educational function due to the lack of teacher knowledge of the different aspects of the number zero.

This study presents the first in-depth analysis of the role of the mathematical picture book in mathematics teaching in the Slovenian classroom. It lays the foundations for the implementation and multifunctional usefulness of the proposed new teaching approach for teaching mathematical concepts at the classroom level.

Keywords: picture book, mathematical picture book, number zero, didactic tool, teaching mathematics

Vloga matematičnih slikanica pri poučevanju koncepta števila nič pri prvošolcih

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Povzetek. Zadnji dve desetletji se kot inovativen učni pristop za podporo učencem pri razumevanju matematičnih konceptov vse bolj uveljavljajo matematične slikanice. Izhodišče prispevka predstavlja matematična slikanica z naslovom "70 češenj", ki obravnava razumevanje koncepta števila nič pri prvošolcih. V teoretičnem delu je predstavljena vloga tega didaktičnega pripomočka za poučevanje matematičnih konceptov, v nadaljevanju pa se osredotočimo na vsebino naše matematične slikanice, ki bralca prek ilustrativne in domišljijske zgodbe seznami z različnimi vidiki obravnave števila nič: kot število na začetku številskega poltraka; kot števka v zapisu mestne vrednosti; kot nevtralen element pri odštevanju in seštevanju; kot simbol za označitev odsotnosti neke količine.

V empiričnem delu raziščemo različne funkcije matematične slikanice pri poučevanju omenjenega koncepta: vzgojna, motivacijska in izobraževalna. Z raziskavo smo želeli ugotoviti, kako učenci 1. razreda prepoznajo vlogo števila nič v različnih kontekstih, kako jo sprejemajo kot didaktični pripomoček ter kakšno je mnenje učiteljev prvih razredov glede uporabnosti izbrane matematične slikanice. Rezultati raziskave so pokazali, da se z uporabo matematične slikanice izboljša učenčevo razumevanje različnih vidikov števila nič. Učenci se pri učni uri z vključeno matematično slikanico počutijo prijetno in prepoznavajo tudi njen vzgojni vidik. V raziskavi smo tudi ugotovili, da učitelji prvega triletja matematično slikanico "70 češenj" prepoznavajo kot uporaben didaktični pripomoček pri poučevanju matematike in tudi drugih učnih predmetov. Prepoznavajo motivacijsko in vzgojno funkcijo tega didaktičnega pripomočka, težave pa se pokažejo pri prepoznavanju izobraževalne funkcije in pomanjkljivem učiteljevem poznavanju različnih vidikov števila nič.

Raziskava predstavlja prvo poglobljeno analizo vloge matematične slikanice za poučevanje matematike v slovenskem učnem prostoru. S tem postavlja temelje za uveljavitev ter večnamensko uporabnost ponujenega novega učnega pristopa za poučevanje matematičnih pojmov na razredni stopnji.

Ključne besede: slikanica, matematična slikanica, število nič, didaktični pripomoček, poučevanje matematike

Games in mathematics classroom: What do teachers and students think about them?

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Abstract. Digital game-based learning (DGBL) has been viewed as an interesting teaching approach that has captured the interest of academics and has become a popular study topic. There is disagreement on the possible advantages of gaming on students' academic achievement, motivation, and skills. According to a number of studies, appropriate teaching practices must be integrated into digital game-based learning in order to successfully increase students' learning outcomes and problem-solving skills.

Given that the use of digital games in secondary education, particularly in mathematics education, is not yet widespread, it is critical to understand what secondary students and teachers think about the learning/teaching experience following a lesson with DGBL.

The European Erasmus+ project GAMMA (GAMe-based learning in MAthematics) focuses particularly on the application of DGBL in mathematics education in the upper secondary school. Throughout the project's duration, seven digital games and eight teaching scenarios were created. Teachers engaged in the project piloted these scenarios in the classroom, and both teachers and students provided feedback on their DGBL experience using a brief questionnaire. This study presents a qualitative and quantitative analysis of post-piloting surveys from students and teachers.

Keywords: digital game-based learning (DGBL), mathematics teachers, students, teaching strategies, digital games, teaching scenarios

Igre u matematičkim učionicama: Što nastavnici i učenici misle o tome?

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Sažetak. Učenje temeljeno na digitalnim igrama (DGBL) smatra se zanimljivim pristupom poučavanju koji sve više zaokuplja akademsku zajednicu i predstavlja popularnu temu za proučavanje. Postoji neslaganje oko prednosti i nedostataka takvog poučavanja na akademska postignuća, motivaciju i vještine učenika. Prema brojnim studijama, odgovarajuće nastavne metode moraju biti integrirane u učenje temeljeno na digitalnim igrama kako bi učenici poboljšali svoje znanje i vještine rješavanja problema.

Uporaba digitalnih igara u srednjoškolskom obrazovanju nije učestala, posebice u poučavanju matematike, stoga je važno istražiti što srednjoškolci i nastavnici misle o takvom poučavanju nakon njegove implementacije u nastavu matematike.

Europski Erasmus+ projekt GAMMA (GAMe-based learning in MAthematics) posebno se fokusira na primjenu DGBL-a u nastavi matematike u srednjim školama. Tijekom trajanja projekta izrađeno je sedam digitalnih igara i osam scenarija poučavanja. Nastavnici uključeni u projekt izrađene scenarije implementirali su u učionici te su i nastavnici i učenici dali povratne informacije o svom DGBL iskustvu pomoću kratkog upitnika. Ova studija prikazuje kvalitativnu i kvantitativnu analizu odgovora učenika i nastavnika nakon provedenog pilotiranja.

Ključne riječi: učenje temeljeno na digitalnim igrama (DGBL), nastavnici matematike, učenici, strategije poučavanja, digitalne igre, scenariji poučavanja

Preschool teacher's planning and implementation of dialogic teaching in dealing with combinatorics situations

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Abstract. The pre-school period for a child, especially if he/she attends kindergarten, represents the entry into the systematic acquisition of mathematical concepts. In this period, these are linked to material reality, but in further learning, the concepts become increasingly detached from it, the concepts becoming abstract mental objects. In addition to some of the most commonly covered topics in the pre-school period (e.g. counting, geometric shapes), research in the field of early mathematics learning emphasises the importance of learning about simple combinatorial situations. Concepts in combinatorics, when represented and conceptualised appropriately, support the child's development of generalisation and systematic thinking skills, while at the same time being quite independent content, i.e. not directly linked to the child's knowledge of other mathematical content. Although the teacher and preschool teacher in the "modern" (as opposed to the "traditional") conception of teaching and learning are systematically being removed from their teaching role, and there is still a concern in the field of pre-school education to "teach the children a lesson", we have decided to highlight or refresh the importance of the preschool teacher's role in the kindergarten. In this paper, we argue that the preschool teacher, as a knowledge holder, has a key role to play in the teaching of mathematics in order for the child to progress in his/her knowledge. One of the hallmarks of good teaching is the thoughtful planning and implementation of dialogue with children in the context of dialogic teaching, which in practice never stands alone, but is intertwined with direct teaching - the preschool teacher also directly transmits certain knowledge, terms, procedures, etc., as part of his/her work. In this paper, we are interested in how the teacher plans and implements dialogue with preschool children when dealing with content from simple combinatorial

situations. We will examine the preschool teacher's dialogue from the point of view of the type and quality of the questions and from the point of view of the preschool teacher's teaching role. The sample will include a group of preschool teachers who have acquired relevant theoretical knowledge on conducting dialogue with preschool children in the context of a course on early mathematics learning, on the basis of which they have planned and implemented activities in kindergarten. Insights into the quality of the preschool teacher's planning and implementation of the mathematical dialogue will be gained by coding and qualitatively analysing transcribed recordings of the preschool teacher's dialogue with children in the process of implementing the mathematical activities, and by comparing the planning of the dialogue and its implementation. We see the contribution of our paper in the fact that, unlike research that mainly examines children's reasoning in solving selected combinatorics problems, our research highlights the role of the kindergarten teacher as an essential actor in the process of children's mathematics learning. Furthermore, in this paper, we will also present examples of qualitative dialogues: both in terms of the way they are carried out and from a mathematical point of view.

Keywords: combinatorics, mathematics, preschool child, dialogic teaching, preschool teacher

Vzgojiteljevo načrtovanje in izvedba dialoškega poučevanja pri obravnavi kombinatoričnih situacij

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Povzetek. Predšolsko obdobje za otroka, še posebej če obiskuje vrtec, predstavlja vstop v sistematično pridobivanje matematičnih pojmov. Ti so v tem obdobju vezani na materialno realnost, v nadaljnjem učenju pa se od nje pojmi vedno bolj odmikajo, pojmi postajajo abstraktni miselni objekti. Poleg nekaterih najbolj pogosto obravnavanih vsebin v predšolskem obdobju (na primer štetje, geometrijske oblike) raziskave s področja zgodnjega učenja matematike poudarjajo pomen učenja o preprostih kombinatoričnih situacijah. Pojmi iz kombinatorike, če so ustrezno reprezentirani in osmišljeni, podpirajo otrokov razvoj sposobnosti posploševanja in sistematičnega mišljenja, hkrati so precej samostojne vsebine, torej niso neposredno vezane na znanje otroka o drugih matematičnih vsebinah Čeprav se učitelja in vzgojitelja v »sodobnem« pojmovanju (za razliko od »tradicionalnega«) poučevanja in učenja sistematično umika iz njegove poučevalne vloge, na področju predšolske vzgoje pa pri nas obstaja še skrb, da bi otroke »pošolali«, smo se odločili izpostaviti oz. ponovno osvežiti pomen vzgojiteljevega delovanja v vrtcu. V prispevku zavzemamo stališče, da ima vzgojitelj, kot nosilec znanja, pri poučevanju matematike ključno vlogo pri tem, da otrok napreduje v znanju. Ena od odlik dobrega poučevanja je premišljeno načrtovanje in izvedba dialoga z otroki v okviru dialoškega poučevanja, ki praktično nikoli ne nastopa samostojno, prepleta se z direktnim poučevanjem – vzgojitelj pri delu namreč določena znanja, termine, postopke ipd. tudi neposredno posreduje. V prispevku nas zanima, kako vzgojitelj načrtuje in izvaja dialog s predšolskimi otroki pri obravnavi vsebin iz preprostih kombinatoričnih situacij. Vzgojiteljev dialog bomo proučevali z vidika vrste in kakovosti vprašanj ter z vidika vzgojiteljeve poučevalne vloge. V vzorec bomo vključili skupino vzgojiteljev, ki so v okviru predmeta o zgodnjem učenju matematike pridobili ustrezna teoretična znanja o vodenju dialoga s predšolskimi

otroki, na osnovi katerega so načrtovali in izvedli aktivnosti v vrtcu. Vpogled v kakovost vzgojiteljevega načrtovanja in izvajanja matematičnega dialoga bomo dobili s kodiranjem in kvalitativno analizo transkribiranih posnetkov vzgojiteljevega vodenja dialoga z otroki v procesu izvajanja matematičnih aktivnosti ter s primerjanjem načrtovanja vodenja dialoga ter njegovo izvedbo. Doprinos našega prispevka vidimo v tem, da za razliko od raziskav, ki v glavnem proučujejo otrokovo razmišljanje pri reševanju izbranih problemov iz kombinatorike, naša raziskava izpostavlja vlogo vzgojitelja kot bistvenega akterja v procesu otrokovega učenja matematike. Nadalje bomo v prispevku predstavili tudi primere kakovostnih dialogov: tako z vidika načina izvajanja kot z matematičnega vidika.

Ključne besede: kombinatorika, matematika, predšolski otrok, dialoško poučevanje, vzgojitelj

On Kárteszi points of a triangle, via three reflections theorem and geometric algebra

To memory of my professor and doctor father, Ferenc Kárteszi (1907-1989)

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Abstract. Let us recall a well-known school task: In the (Euclidean E^2) plane of a triangle ABC we draw regular triangles outward on sides of ABC, say ABC^-, BCA^-, CAB^-, respectively. Prove that the segments AA^-, BB^-, CC^- intersect each other in a point K, that is the isogonal point of ABC and the distance sum AK + BK + CK is minimal for K among all points of the plane.

Professor Kárteszi noticed that instead of regular triangles we can draw isosceles ones with all equal base angles, and the above K (called Kárteszi point) exists also in the Bolyai–Lobachevsky hyperbolic plane H^2 (in the sphere S^2 as well, see Kálmán [1]), the orthocentre, barycentre are specific cases. There is a more general extremum problem of I. M. Yaglom [6] (problem 83, modified here):

In the plane (E^2) of a given triangle ABC find a point K such that the quantity $\alpha KA + \beta KB + \gamma KC$, where α, β, γ are given positive numbers, has the smallest possible value.

This problem leads to a more general triangle configuration and to an analogous extremal point K. Moreover, as a new result of this paper, an extension onto "absolute plane" (S^2, E^2, H^2, M^2 Minkowski plane, G^2 Galilei (or isotropic) plane) can be formulated and solved by three reflections theorem [3][5] and geometric (Grassmann–Clifford type) algebra [3]. Open problems arise as well. By this we want to follow F. Kárteszi's didactical credo (see also his wonderful book [2] of great international success):

Start with a natural, elementary, visually well understandable task! Then follow the manipulations, tools, new mathematical concepts, the technical machinery; then the solution, occasional theory, further applications, extensions, …

Keywords: triangle geometry, three reflections theorem, geometric (Grassmann-Clifford type) algebra, absolute geometry, problem-solving by similarity transformation

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A háromszög Kárteszi pontjairól, a három tükrözés tétele és geometriai algebra segítségével

Kárteszi Ferenc (1907-1989), professzor úr, tanárom, doktori témavezetőm emlékére

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Absztrakt. Idézzük fel az alábbi ismert feladatot: Az ABC háromszög (euklideszi E2) síkjában szabályos háromszögeket rajzolunk kifelé az oldalakra, mondjuk ABC^{-}, BCA^{-}, CAB^{-} jelöléssel. Bizonyítsuk be, hogy az AA^{-}, BB^{-}, CC^{-} szakaszok egy K pontban metszik egymást. Ez a K pont az ABC un. izogonális pontja (melyből az oldalak egyforma 120° szögben látszanak), és melyre az AK + BK + CK távolságösszeg a legkisebb, a sík bármely K-tól különböző pontját is tekintjük.

Kárteszi professzor észrevette, hogy ez a K pont akkor is létrejön, ha szabályos háromszögek helyett egyenlő alapszögű egyenlő szárú háromszögeket rajzolunk ABC oldalaira kifelé, továbbá ez a Bolyai-Lobacsevszkij-féle hiperbolikus geometriában (sőt az S2 (gömbfelületi) szférikus síkon, lásd még erről Kálmán Attila [1] könyvét) is igaz. Ezért is javaslom a Kárteszi pont elnevezést. A magasságpont, súlypont speciális esetek lesznek. Rokonságban van a témával I. M. Yaglom (Jaglom) [5] Geometriai transzformációk II könyvének 83. szélsőérték feladata (módosított jelölésekkel):

Egy adott ABC háromszög (E^2) síkjában keressük meg azt a K pontot, melyre az $\alpha KA + \beta KB + \gamma KC$ mennyiség a lehető legkisebb, ahol α, β, γ adott pozitív számok.

Ez a feladat általánosabb háromszög alakzatokhoz és analóg extremális K ponthoz vezet. Továbbá, és ez új eredményünk a dolgozatban, a Bolyai János-féle "abszolút síkra" (az S^2 gömbre, E^2, H^2 síkokra, az M^2 Minkowski és a G^2 Galilei (vagy izotróp) síkokra egyaránt) érvényes módon lehet megfogalmazni és megoldani a problémát a három tükrözés tétele [3][5] és az un. geometriai (Grassmann-Clifford típusú) algebra segítségével [4]. Így további nyitott kérdések is felvetődnek. Ezzel

követni kívánjuk Kárteszi Ferenc módszertani hitvallását (lásd pl. nagy nemzetközi sikerű [2] csodálatos könyvét):

Induljunk ki egy természetes, elemi, szemléletesen is jól érthető feladatból. Ezután következhetnek a próbálkozások, matematikai kísérletek, eszközök és fogalmak kialakítása, majd a megoldás, esetleges elmélet, további alkalmazások és általánosítások, …

Kulszavak: háromszög geometria, három tükrözés tétele, geometriai (Grassmann-Clifford típusú) algebra, abszolút geometria, probléma-megoldás hasonlósági transzformációval.

Advanced education methods and application of mathematical knowledge in classes with advanced programmers

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Abstract. On a sample of 12 fifth-grade mathematicians and programmers from elementary schools in Osijek and its surroundings, selected by teachers as the best from their schools, a study was conducted on the correlation between the ability to solve mathematical and programming tasks. For this purpose, at the beginning and the end of the education, the students took tests to determine their initial programming skills, as well as mathematical thinking, and their progress at the end of the school year. During the education, the educators monitored each student's motivation for solving problematic programming tasks considering the knowledge of the mathematical background that was in that task. The aim of this research is to determine if fifth-grade students, who have good mathematical skills, can accurately and precisely model a problem while solving programming tasks.

The type of educational research and conclusions are given in the paper.

Keywords: advanced education methods, logical-mathematical thinking, programming, problem tasks, problem modelling

Napredne metode edukacije i primjena matematičkog znanja u nastavi s naprednim programerima

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Sažetak. Na uzorku od 12 matematičara i programera petih razreda osnovnih škola grada Osijeka i okolice, odabranih od strane nastavnika kao ponajboljih iz svojih škola, napravljeno je istraživanje o korelaciji sposobnosti rješavanja matematičkih i programerskih zadataka. U tu svrhu, na početku i na kraju edukacije učenici su rješavali ispit kako bi se utvrdilo njihovo početno znanje iz programiranja, ali i matematičkog razmišljanja te njihovo napredovanje na kraju školske godine. Tijekom edukacije, edukatori su pratili kod svakog polaznika njegovu motivaciju za rješavanjem problemskih programerskih zadataka s obzirom na poznavanje matematičke podloge koja se nalazila u tom zadatku. Cilj istraživanja je utvrditi mogu li učenici petih razreda, koji su ujedno i dobri matematičari, točno i precizno modelirati problem prilikom rješavanja programskih zadataka.

Oblik edukacije istraživanja i zaključci su dani u članku.

Ključne riječi: napredne metode edukacije, logičko-matematičko razmišljanje, programiranje, problemski zadaci, modeliranje problema

Mathematics teachers' continuous professional development – Reflections on Lifelong Learning Program "Enactive Learning in Mathematics"

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Abstract. The Lifelong Learning Program "Enactive Learning in Mathematics" was implemented for the first time at the Faculty of Mathematics, University of Rijeka, in the winter semester 2022/2023. After the completion of the program, interviews were conducted with mathematics teachers who had participated in the program. In addition to evaluating the program itself, we asked teachers about their motivation to participate in lifelong learning programs and the support they received. This paper provides reflections on the interviews conducted and the program implemented.

Keywords: professional development, mathematics teacher, lifelong learning program, enactive learning in mathematics, education policy

Kontinuirani profesionalni razvoj nastavnika matematike – Refleksije na Program cjeloživotnog obrazovanja "Iskustveno učenje matematike"

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Sažetak. Program cjeloživotnog obrazovanja "Iskustveno učenje matematike" po prvi je puta održan na Fakultetu za matematiku Sveučilišta u Rijeci u zimskom semestru 2022./2023. akademske godine. Po njegovom završetku provedeni su razgovori s nastavnicima matematike koji su sudjelovali u programu. Osim osvrta na sam program, nastavnike smo pitali koja je njihova motivacija za sudjelovanje u programima cjeloživotnog obrazovanja i koliku podršku imaju u tome. Ovaj rad sadrži refleksije na provedene razgovore i izvedeni program.

Ključne riječi: profesionalni razvoj, nastavnik matematike, program cjeloživotnog obrazovanja, iskustveno učenje matematike, obrazovna politika

STEAM-BOX project

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Abstract. In this presentation, I would like to talk about the "STEAM-BOX": Courses, Tools, Resources for Teachers Erasmus+ project.

Nowadays, the transdisciplinary integration of STEAM (science, technology, engineering, arts, mathematics) into learning processes is becoming increasingly important. STEAM includes new skills and key competencies that we need in the 21st century. The growing importance of this field is reflected in the fact that its education has been the subject of a number of recent studies and education policy reports. However, there are still gaps in implementation.

Our "STEAM-BOX" project aims to support STEAM education, to create intellectual products to increase students' competencies as much as possible, and to develop teaching aids for teachers.

We have created a GeoGebra gamified platform, organized an online course for teachers, produced teaching aids and published an e-book on STEAM education. We involve primary and secondary school teachers and students in testing the exercises and gamification elements.

After the general introduction, I will briefly present two modules from the online course for teachers and some exercises to illustrate the STEAM approach to teaching we have developed.

Keywords: STEAM, gamification, GeoGebra, transdisciplinary integration, education

STEAM-BOX projektünk

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Absztrakt. Az előadásban a STEAM-BOX oktatási módszerek, források és eszközök tanárok számára Erasmus+ projektről szeretnék beszámolni.

Napjainkban egyre fontosabbá válik az integrált oktatás, ezen belül a STEAM (tudomány, technológia, mérnöki tudományok, művészetek, matematika) területek transzdiszciplináris beépítése a tanulási folyamatokba. A STEAM olyan új készségeket és kulcskompetenciákat foglal magába, amelyekre szükségünk van a 21. században. A terület növekvő fontosságát jelzi az is, hogy oktatásának kérdése számos friss tanulmányban, oktatáspolitikai jelentésben szerepel. Ennek ellenére a végrehajtás terén még mindig hiányosságok vannak.

"STEAM-BOX" projektünk célja a STEAM oktatás támogatása, a tanulók kompetenciáinak minél hatékonyabb növelése érdekében szellemi termékek létrehozása, oktatási segédanyagok fejlesztése tanároknak.

A projekt keretében létrehoztunk egy játékosított GeoGebra platformot, online kurzust szerveztünk tanároknak, oktatási segédanyagokat készítettünk és egy e-bookot jelentettünk meg a STEAM szemléletű oktatásról. A feladatok, és a játékosítási elemek tesztelésébe általános és középiskolai tanárokat, tanulókat vonunk be.

Az általános ismertető után röviden bemutatnék két modult a tanárok online kurzusából és néhány feladatot, hogy illusztráljam a STEAM szemléletű oktatás általunk kidolgozott módszerét.

Kulcsszavak: STEAM, játékosítás, GeoGebra, transzdiszciplináris integráció, oktatás

The potential of number theory in the development of mathematical thinking

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Abstract. Number theory is a small part of the Hungarian secondary school curriculum. Number theory is an area of mathematics where the ideas used are different in many ways, and where the types of problems are varied. Students can encounter a wide range of problems, from simple basic problems to Olympic competition problems. This is why we started to investigate how number theory tasks can affect the development of students' thinking and general mathematical reasoning in public schools. We designed an experiment that links the learning of number theory and the solving of number theory problems with general mathematical problem solving skills.

Seventh and eighth grade students participated in the experiment. The students in the experiment were divided into two groups, an experimental group and a control group. The students in the experimental group were given a number theory task at the beginning of each lesson, while the students in the other group were given a task related to the regular curriculum. The effectiveness of the experiment was tested on both groups by regular curriculum assessments and by levelled assessments in which number theory was not included.

Keywords: problem solving, number theory, secondary school, Case Study, classroom routine

A matematikai gondolkodás fejlesztése számelméleti feladatokkal

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Absztrakt. A magyarországi középiskolai tananyagban a számelmélet témaköre kis súllyal szerepel. A számelmélet a matematikának az a területe, ahol a legkülönbözőbb ötletekkel, legváltozatosabb feladattípusokkal találkozunk az egyszerű órai feladatoktól az olimpiai versenyfeladatokig. Ezért kezdtük el vizsgálni, hogyan hathatnak a számelmélettel kapcsolatos feladatok a közoktatásban tanuló diákok gondolkodásának fejlődésére, általános matematikai gondolkodására. Egy kísérletet terveztünk, amely a számelmélettel való ismerkedést, számelméleti feladatok megoldását összeköti az általános matematikai problémamegoldó képességgel. A kísérletben hetedik és nyolcadik osztályos tanulók vettek részt. A kísérletben szereplő diákokat két részre osztottuk, egy kísérleti és egy kontroll csoportra. A kísérleti csoport diákjai minden óra elején egy számelmélettel kapcsolatos feladatot oldottak meg, a másik csoport tagjai a reguláris tananyaggal kapcsolatosan kaptak feladatot. A kísérlet eredményességét a reguláris tananyaggal kapcsolatos dolgozatokkal, és olyan szintfelmérőkkel vizsgáltuk, melyekben a számelmélet témaköre nem szerepelt.

Kulcsszavak: problémamegoldás, számelmélet, középiskola, esettanulmány, órai rutin

Discipline of noticing – The case of three university teachers

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Abstract. Classroom practice greatly relies on the experiences and observations of the teachers. We notice what students do, how they respond, we reflect on what is presented or done in the lectures or teaching materials in relation to learning outcomes. In the classroom, we constantly notice, for example, we notice when students do not understand the lesson, we notice that students lose concentration, so during the lectures we give more attention to different methods and content of the lecture. Noticing these critical moments when we make, or might make, choices, and the disposition of alternative responses at hand, provides a way of informing and developing our teaching practice.

John Mason introduced the discipline of noticing in 2002 as a systematic "method of sensitising oneself to notice possibilities for action" and change in the future through noticing. This methodology can be applied in the process of a teacher's professional development. The process of becoming "more expert in the field" can be considered as a process of "broadening and deepening sensitivities to notice and to act" in the moment.

In the last 20 years, there has been a significant amount of research dealing with the topic of teacher noticing. However, there aren't many papers in which Mason's methodology is used. In this talk, we report on the experiences of three university teachers of mathematics from different faculties who implemented discipline of noticing in their own practice within the period of six months.

Keywords: teacher noticing, reflection-in-action, professional development, university mathematics, researching from the inside, using reports

Disciplinirano opažanje tri sveučilišna nastavnika

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Sažetak. Svaki čin poučavanja ovisi o promatranju, primjećujemo što studenti rade, kako reagiraju, promišljamo o onome što se prezentira ili radi na predavanjima ili nastavnim materijalima u odnosu na ishode učenja. U nastavi stalno primjećujemo, primjerice, primjećujemo kada učenici ne razumiju nastavu, primjećujemo da učenici gube koncentraciju, pa tijekom predavanja više pozornosti pridajemo različitim metodama i sadržajima predavanja. Primjećivanje ovih kritičnih trenutaka kada pravimo, ili bismo mogli napraviti, različite izbore te raspolaganje alternativnim odgovorima u danom trenutku, pruža način informiranja i razvoja naše nastavne prakse.

John Mason u 2002. godini uvodi disciplinirano opažanje kao sustavnu metodu senzibiliziranja za opažanje mogućnosti djelovanja i promjene u budućnosti kroz opažanje. Ova metodologija može se primijeniti u procesu stručnog usavršavanja nastavnika. Proces postajanja "većim stručnjakom u području" može se smatrati procesom širenja i produbljivanja osjetljivosti za uočavanje i djelovanje u trenutku.

U posljednjih 20-ak godina postoji značajan broj istraživanja koja se bave temom učiteljskog opažanja. Međutim, nema mnogo radova u kojima se koristi Masonova metodologija. U ovom izlaganju donosimo iskustva trojice sveučilišnih nastavnika matematike s različitih fakulteta koji su u razdoblju od šest mjeseci implementirali disciplinu opažanja u vlastitoj praksi.

Ključne riječi: učiteljsko opažanje, refleksija-u-akciji, profesionalni razvoj, sveučilišna matematika, istraživanje iznutra, korištenje izvještaja

Fermi problems as a mathematical modelling activity in secondary education

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Abstract. Mathematical modelling is an extremely important but complex process, and it is sometimes difficult to find room for it in crowded curricula. The teacher plays an important role in selecting tasks and matching them to students. When choosing modelling tasks, one can opt for closed task types that focus on specific knowledge and make it easier for the teacher to direct and control them, or open-ended tasks that are more cognitively challenging for students. Open-ended modelling tasks have the advantage of allowing students to explore the nature of the observed phenomenon and create an effective mathematical representation that enables them to better understand reality. However, it is difficult for the teacher to anticipate all the possible interpretations, strategies, and mathematical concepts that students will use in creating their mathematical model. It is also necessary to ensure that students have enough time to explore open-ended problems. Therefore, such problems are rarely addressed in class and are more often assigned as independent or group projects as homework.

This paper shows how a group of seventeen-year-old students from a Croatian high school solved two Fermi problems that required estimating the number of people on a given surface. The main goal was to investigate the didactic potential of a set of Fermi estimation problems as an open mathematical modelling activity in secondary education. The models that students constructed in individual and small group work are characterized, and how the models evolved with respect to the constraints imposed by the realistic context in the different tasks is analyzed.

Keywords: mathematical modelling, secondary education, open task, Fermi problems, students' strategies

Fermijevi problemi procjena kao aktivnost matematičkog modeliranja u srednjoškolskom obrazovanju

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Sažetak. Matematičko modeliranje iznimno je važan, ali složen proces, te je u prenatrpanim nastavnim planovima i programima ponekad teško naći prostora za njega. Nastavnik ima bitnu ulogu odabira problema i prilagodbe problema svojim učenicima. Prilikom odabira problema modeliranja moguće je odabrati zatvorene tipove problema koji su usmjereni na specifična znanja stoga ih nastavnik može lakše usmjeravati i kontrolirati ili otvorene probleme koji su učenicima kognitivno zahtjevniji. Prednost otvorenih tipova problema modeliranja je da omogućuju učenicima istraživanje prirode promatranog fenomena te kreiranje učinkovite matematičke reprezentacije koja im omogućuje da bolje razumiju stvarnost. Međutim, nastavniku je teško predvidjeti sve moguće interpretacije, strategije i matematičke koncepte koje će učenici primijeniti prilikom stvaranja svog matematičkog modela. Uz to, potrebno je osigurati dovoljno vremena učenicima za istraživanje otvorenog tipa problema, stoga su takvi tipovi problema najrjeđe zastupljeni na nastavi i češće se zadaju kao samostalni ili grupni projekti za domaću zadaću.

U ovom radu bit će prikazano kako grupa sedamnaestogodišnjaka, polaznika srednje škole u Hrvatskoj, rješava dva Fermijeva problema koji zahtijevaju procjenu broja ljudi na zadanoj površini. Osnovna svrha bila je istražiti didaktički potencijal niza Fermijevih problema procjena kao aktivnosti otvorenog matematičkog modeliranja u srednjoškolskom obrazovanju. Karakterizirat će se modeli koje su učenici konstruirali dok su radili individualno i u malim grupama te će se analizirati kako su modeli evoluirali s obzirom na restrikcije nametnute realističnim kontekstom u različitim zadacima.

Ključne riječi: matematičko modeliranje, srednjoškolsko obrazovanje, zadatak otvorenog tipa, Fermijevi problemi procjena, strategije učenika

Superficial strategies in solving compare-combine problems

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Abstract. Compare and combine problems are used in elementary mathematics as a part of standard teaching practice. Their integration enables creation of various types of mathematical problems with different structure and level of difficulty. One of the main obstacles in solving word problems is the use of superficial strategies in which students directly translate words they have recognized as entities and relations into mathematical operations and expressions, without understanding the situational model of the problem. The aim of this paper is to investigate the use of these strategies in solving integrated compare-combine problems. For this purpose, we posed problems with varying correspondences between entity key words and relations given in the text of the problems. One hundred and thirty-four students participated in the study by solving paper and pencil test. Forty-four students were in 2nd grade (7.5–8.5 years old students), 48 in 4th grade, and 42 in 6th grade. Results showed that students did not have different achievement on problems with different correspondence between entity key words and relations. The superficial approach they used most often was in identifying relational terms (mathematical operations). As expected, there were differences in achievement and in nature of mistakes regarding students' level of education (2nd, 4th or 6th grade).

Keywords: compare problems, combine problems, problem solving, strategies, word problems

Površinske strategije rešavanja problema kombinovanja-poređenja

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Sažetak. U nastavi matematike za niže razrede osnovne škole, problemi poređenja i kombinovanja se često koriste kao deo uobičajene prakse. Integracija ovih problema omogućava kreiranje mnogih matematičkih problema različite strukture i složenosti. Dominantna teškoća u rešavanju tekstualnih problema je upotreba površinskih strategija u kojima učenici direktno prevode reči koje su prepoznali kao entitete i relacije u matematičke operacije i izraze, bez razumevanja situacionog modela problema. Cilj ovog rada je da istraži upotrebu strategija u rešavanju integrisanih problema poređenja-kombinovanja. U tu svrhu, osmislili smo probleme sa različitim korespodencijama između ključnih reči entiteta i relacija koje su date u tekstu problema. Na testiranju pismenog tipa učestvovalo je 144 učenika, i to 44 učenika drugog razreda (7.5–8.5 godina starosti), 48 učenika četvrtog razreda i 42 učenika šestog razreda. Rezulati su pokazali da učenici nisu imali različita postignuća na problemima sa različitom korespondencijom između ključnih reči entiteta i relacija. Najčešće korišćena površinska strategija je bila u identifikaciji relacionih termina (matematičkih operacija). Očekivano, postojale su razlike u postignuću na ovim zadacima među učenicima drugog, četvrtog i šestog razreda.

Ključne riječi: problemi poređenja, problemi kombinovanja, rešavanje problema, strategije, tekstualni problemi

The *traditional–contemporary* construct in relation to the learning paradigms in the discipline of didactics of mathematics

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Abstract. In Slovenia, both in the general pedagogical field and in the field of teaching and learning mathematics, a distinction was made between so-called *traditional* and *contemporary* teaching, which introduced various dichotomies into the pedagogical domain (e.g. passive and active learning, transfer of knowledge and independent learning, abstract and concrete mathematics teaching). The problematic nature of this phenomenon has been addressed in the past by some authors who considered such a distinction controversial. With the recognition that certain dichotomies are problematic, important steps have been taken to overcome them. This paper continues this process of overcoming by considering the *traditional–contemporary* construct in relation to some of the learning paradigms that have prevailed in the history of didactics of mathematics. After an initial definition of the *traditional–contemporary* construct and an outline of the emergence of the discipline of didactics of mathematics, we describe some of the most influential learning paradigms that have had a significant impact on the design and implementation of disciplinary research. We point out the problem of committing to a single learning paradigm and discuss the implications of adopting the *traditional–contemporary* construct. We conclude this paper with concrete examples from the field of teaching and learning mathematics to show how the ideological discourse of the *traditional–contemporary* construct can be overcome.

Keywords: *traditional–contemporary* construct, learning paradigms, overcoming ideological discourse, teaching and learning mathematics

Konstrukt tradicionalno–sodobno v navezavi s paradigmami učenja na področju discipline didaktike matematike

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Povzetek. V Sloveniji se je na splošnem pedagoškem področju, kot tudi na področju poučevanja in učenja matematike, uveljavilo razlikovanje med t. i. *tradicionalnim* in *sodobnim* poučevanjem, ki je v pedagoški prostor vneslo različne dihotomije (npr. pasivno in aktivno učenje, prenos znanja in samostojno učenje, abstraktno in konkretno poučevanje matematike). Problematiko obstoječega fenomena so v preteklosti že obravnavali nekateri avtorji in nakazali, da je takšno razlikovanje sporno. S prepoznavo problematičnosti posameznih dihotomij, so bili narejeni pomembni koraki k njihovi premostitvi. S pričujočim prispevkom želimo z obravnavo konstrukta *tradicionalno–sodobno* v navezavi na nekatere paradigme učenja, ki so se skozi zgodovino uveljavile na področju discipline didaktike matematike, proces preseganja nadaljevati. Po začetni opredelitvi konstrukta *tradicionalno-sodobno* in orisu nastanka discipline didaktike matematike, opišemo nekatere najvplivnejše paradigme učenja, ki so pomembno vplivale na snovanje in izvedbo disciplinarnih raziskav. Izpostavimo problematičnost prizadevanj po uveljavitvi zgolj ene izmed njih in razpravljamo o posledicah sprejemanja konstrukta *tradicionalno-sodobno*. V zaključku na konkretnih primerih s področja poučevanja in učenja matematike prikažemo, kako lahko ideološki diskurz konstrukta *tradicionalno-sodobno* presežemo.

Ključne besede: konstrukt *tradicionalno–sodobno*, paradigme učenja, preseganje ideološkega diskurza, poučevanje in učenje matematike

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