

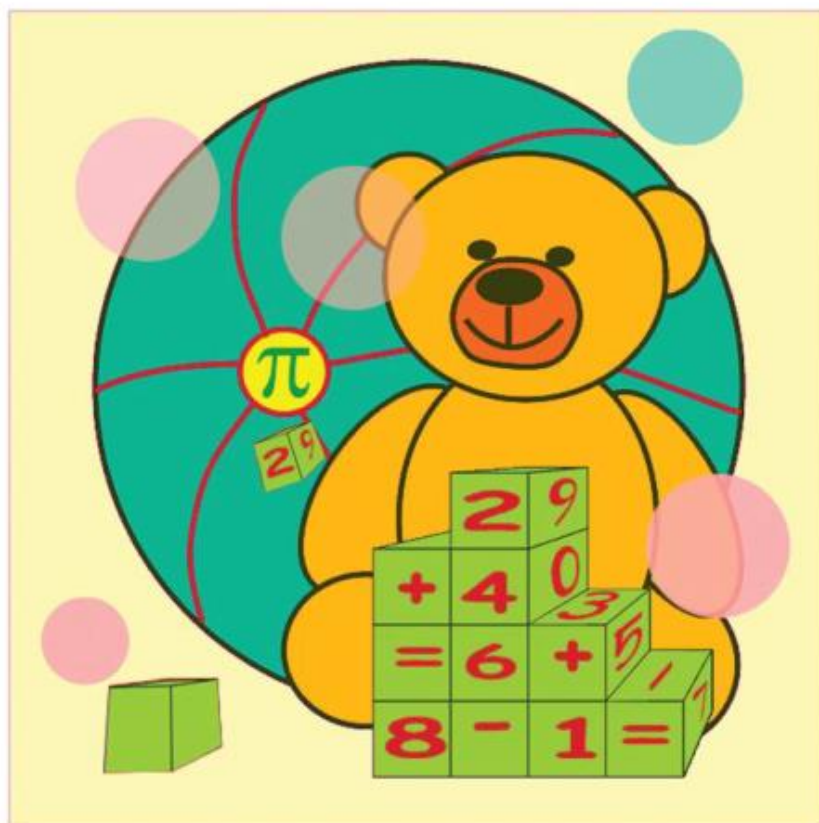
Josip Juraj Strossmayer University of Osijek



Faculty of Education



Department of Mathematics



**The 5<sup>th</sup> International Scientific Colloquium  
MATHEMATICS AND CHILDREN**

# **PROGRAM & BOOK OF ABSTRACTS**

**Editors:**  
**Zdenka Kolar-Begović**  
**Ružica Kolar-Šuper**  
**Ivana Đurđević Babić**

**Croatia, Osijek, May 29-30, 2015**

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# PROGRAM



## FRIDAY, May 29

8.00-9.00 Registration

9.00-9.30 OPENING

### Morning Session

9.30-10.15	Plenary lecture Anders Hast <i>Mathematics+Computer Science=True</i>	<i>p. 17</i>
10.15-10.30	Marijana Zekić-Sušac, Ivana Đurđević Babić <i>Discovering patterns of student behaviour in e-learning environment</i>	<i>p. 18</i>
10.30-10.45	Tatjana Hodnik Čadež, Vida Manfreda Kolar <i>Understanding of mathematically gifted students' approaches to problem solving</i>	<i>p. 10</i>
10.45-11.00	Dušan Mundar, Zlatko Erjavec <i>Pass rates in mathematical courses: relationship with the state matura exams scores and high school grades</i>	<i>p. 23</i>
11.00-11.15	Coffee Break	

### IN MEMORIAM Dr. Margita Pavleković

11.15-12.15	Mirko Polonijo Margita Pavleković i <i>Matematika i dijete</i>	<i>p. 1</i>
	Emil Molnár, Tatjana Hodnik Čadež, Željka Milin Šipuš, Aleksandra Čižmešija, Marijana Zekić-Sušac	

12.15-13.30 Laying flowers on Professor Pavleković's grave

13.30-15.00 Lunch Break (Restaurant *Campus*)

### Afternoon Session

15.00-15.20	Invited lecture Vladimir Volenec <i>Is any angle a right angle?</i>	p. 32
15.20-15.35	Zdenka Kolar-Begović, Ružica Kolar- Šuper, Vladimir Volenec <i>An interesting analogy of Kimberling-Yff's problem</i>	p. 33
15.35-15.50	Doris Dumičić Danilović, Sanja Rukavina <i>Preservice mathematics teachers' problem solving processes when working on two nonroutine geometry problems</i>	p. 28
15.50-16.05	Karmelita Pjanić, Sanela Nesimović <i>Tendencies in identifying geometric shapes observed in photos of real objects – case of students of primary education</i>	p. 29
16.05-16.20	Karmelita Pjanić, Edin Liđan <i>Graphical representations in teaching GCF and LCM</i>	p. 16
16.20-16.45	Coffee Break	

16.45-17.00	Ljerka Jukić Matić, Ana Mirković Moguš, Marija Kristek <i>Pre-service teachers and statistics: an empirical study about attitudes and reasoning</i>	p. 34
17.00-17.15	Aleksandra Čižmešija, Željka Milin Šipuš <i>Beliefs about mathematics and mathematics teaching of students in mathematics education programme at the Department of Mathematics, University of Zagreb</i>	p. 35
17.15-17.30	Ljerka Jukić Matić, Dubravka Glasnović Gracin <i>How Croatian mathematics teachers organize their teaching in lower secondary classrooms: differences according to the initial education</i>	p. 37
17.30-17.45	Goran Trupčević, Anđa Valent <i>Structures of Croatian mathematics textbooks</i>	p. 38
17.45-18.00	Edith Debrenti <i>Word problems in mathematics teaching</i>	p. 14

18.30      Short sightseenig tour

20.00      Conference Dinner (Hotel Osijek)

## SATURDAY, May 30

### Morning Session

9.00-9.30	Invited lecture Emil Molnár, István Prok, Jenő Szirmai <i>Visual mathematics and geometry, the “final” step: projective geometry through linear algebra</i>	p. 31
9.30-9.45	Ivana Đurđević Babić, Anita Marjanović <i>Classification trees in detecting students’ motivation for maths from their ICT and Facebook use</i>	p. 19
9.45-10.00	Josipa Matotek <i>Using Moodle in teaching mathematics in Croatian education system</i>	p. 20
10.00-10.15	Karolina Dobi Barišić <i>Future teachers’ perception on the application of ICT in the process of assessment and feedback</i>	p. 21
10.15-10.30	Vjekoslav Galzina <i>Lotfi A. Zadeh: one man STEM</i>	p. 22
10.30-11.00	Coffee Break	

11.00-11.15	Maja Cindrić, Irena Mišurac <i>Contemporary methods of teaching mathematics – the discovering algorithm method. Algorithm for fraction division</i>	p. 12
11.15-11.30	Zoran Horvat <i>Issues in contemporary teaching of mathematics and teacher competencies</i>	p. 25
11.30-11.45	Željko Rački, Ana Katalenić, Željko Gregorović <i>Self-reported creativity of primary school teachers and students of teacher studies in diverse domains, and implications of creativity relationships to teaching mathematics in the primary school</i>	p. 36
11.45-12.00	Ksenija Romstein, Stanislava Irović, Mira Vego <i>Teaching Mathematics in early education: current issues in classrooms</i>	p. 26
12.00-12.15	Sead Rešić, Ivana Kovačević <i>Approaches to teaching mathematics in lower primary education</i>	p. 24

12.15

CLOSING





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### **Margita Pavleković i Matematika i dijete**

Ovogodišnji, svibanjski peti međunarodni znanstveni skup *Matematika i dijete* (*International Scientific Colloquium "Mathematics and Children"*) održava se po prvi puta bez njegove idejne začetnice, potpune oblikovateljice, organizatorice i neumorne voditeljice pokojne profesorice dr.sc. Margite Pavleković (1948.-2014.).

Skup je pokrenula 2007. godine i u dvogodišnjim razmacima, u četiri navrata, uspješno organizirala, okupivši svaki puta u Osijeku respektabilan broj predavača iz niza zemalja. Profesorica Margita Pavleković je uredila i popratila uvodnim tekstovima zbornike, odnosno tematske monografije vezane uz održane kolokvije i na njima izložena predavanja. Prve tri je uredila samostalno, a četvrtu u suradnji s dr.sc. Zdenkom Kolar-Begović i dr.sc. Ružicom Kolar-Šuper. Zbornici/monografije sveukupno obasižu oko 1500 stranica s preko 100 članaka i u potpunosti su dostupni preko internetskih stranica osječkog Fakulteta za odgojne i obrazovne znanosti:

[http://web.foozoh.hr/images/dokumenti/skupovi-publikacije/01\\_Zbornik\\_matematika\\_dijete.pdf](http://web.foozoh.hr/images/dokumenti/skupovi-publikacije/01_Zbornik_matematika_dijete.pdf)

[http://web.foozoh.hr/images/dokumenti/skupovi-publikacije/02\\_Learning\\_Outcomes-Monography.pdf](http://web.foozoh.hr/images/dokumenti/skupovi-publikacije/02_Learning_Outcomes-Monography.pdf)

[http://web.foozoh.hr/images/dokumenti/skupovi-publikacije/03\\_The\\_Math\\_Teacher-Monography.pdf](http://web.foozoh.hr/images/dokumenti/skupovi-publikacije/03_The_Math_Teacher-Monography.pdf)

<http://files.eric.ed.gov/fulltext/ED542544.pdf>

Dosadašnji susreti su održani u Osijeku 13.04.2007. (tema: *Kako učiti i poučavati matematiku*), 24.-25.04.2009. (*Ishodi učenja*), 18.-19.03.2011. (*Učitelj matematike*), 19.-20.04.2013. (*Poučavanje matematike za budućnost*). Organizatori skupa su svaki puta bili osječki Učiteljski fakultet i Odjel za matematiku Sveučilišta J. J. Strossmayer. Prikaze i detaljne opise tih susreta dr.sc. Margita Pavleković objavila je u stručno-metodičkom časopisu *Matematika i škola* iz Zagreba:

Matematika i škola (2006/2007) Vol.8, No. 40, 227-229

Matematika i škola (2008/2009) Vol.10, No. 50, 237-238

Matematika i škola (2010/2011) Vol.12, No. 59, 187-189

Matematika i škola (2012/2013) Vol.14, No. 70, 232-234

Trenutak pokretanja kolokvija/ znanstvenog skupa *Matematika i dijete* bio je osobito važan i od iznimnoga značaja za matematički kadar na učiteljskim fakultetima u Hrvatskoj prije svega zato što su u to vrijeme učiteljski studiji iz stručnih prerasli u sveučilišne studije. Nadalje, bio je to dodatni korak i novi poticaj u oživljavanju doktorskih studija iz

matematike, smjer edukacijska matematika. Na okruglom stolu upriličenom u okviru prvog kolokvija imenovano je povjerenstvo u sastavu: dr.sc. Aleksandra Čižmešija, dr.sc. Željka Milin Šipuš, dr.sc. Margita Pavleković i dr.sc. Sanja Varošaneć “koje treba obaviti sve formalne korake prema službenom zahtjevu za odobrenje novoga smjera na doktorskim studijima iz matematike – smjera edukacijske matematike, po ugledu na već postojeće u Europi i Americi”. Dvije godine kasnije: “Na okruglom stolu članice povjerenstva ... izvjestile su o malim, ali značajnim pomacima prema zaživljavanju novoga smjera na doktorskim studijima iz matematike – smjera edukacijske matematike. (...) Od akademske 2008./09. godine zainteresirani studenti doktorskih studija na Matematičkom odjelu PMF-a u Zagrebu upisali su seminar iz Metodike nastave matematike. Nositeljice seminara su članice spomenutoga povjerenstva.” Dvije godine kasnije urednica zbornika piše: “Problem cjeloživotne izobrazbe i znanstvenoga napredovanja učitelja (matematike) u području edukacijske matematike još uvijek nije riješen na način kojim bi se u akademskoj zajednici svakom učitelju pružila realna prilika da znanstveno napreduje.” Pišući o četvrtom kolokviju *Matematika i dijete* održanom 2013., u časopisu *Matematika i škola* se navodi: “... pokrenut je u Osijeku 2007. godine na inicijativu Margite Pavleković i njezinih suradnika. Skup je pokrenut u trenutku kada se u Hrvatskoj akademskoj zajednici nastoji osvijestiti važnost Metodike matematike te nastoji operacionalizirati napredovanje onih koji se znanstveno i stručno bave matematičkom edukacijom. S druge strane, niti u Europi niti izvan nje do tada nije zabilježen skup kojemu središnju temu čini matematička izobrazba na svim dobnim razinama - predškolska djeca, učenici osnovnih i srednjih škola, studenti, posebno učiteljskih odnosno nastavničkih fakulteta. Naslovom skupa namjera je bila privući domaće i strane matematičare koji istražuju u području edukacijske matematike u najširem smislu te riječi, a koji na specijaliziranim znanstvenim skupovima iz raznih matematičkih grana nisu pronalazili dovoljno prilika da o tome izlažu. Također se naslovom skupa nastojala istaknuti potreba popularizacije matematike među najmlađima te neupitna i presudna važnost matematičke izobrazbe od rane dječje dobi. Organizacija takvoga skupa svesrdno je prihvaćena i potpomognuta od strane akademske i stručne zajednice. ... Postignuti su značajni pomaci u mogućnostima znanstvenoga napredovanja pojedinaca koji istražuju u području edukacijske matematike, što je bio inicijalni cilj pokretanja skupa *Matematika i dijete*.”

Upravo je u okviru znanstvenog kolokvija Matematika i dijete, 2009. u prepunoj svečanoj dvorani Učiteljskoga fakulteta u Osijeku, promovirana važna znanstvena studija dr.sc. Margite Pavleković *Matematika i nadareni učenici – razvoj kurikula na učiteljskim studijima za prepoznavanje, izobrazbu i podršku darovitih učenika*, objavljena kod uspješnog zagrebačkog izdavača matematičkih knjiga i časopisa „Element“.

Djelo podastire značajan iskorak u mogućnosti kvalitetnijeg obrazovanja studenata učiteljskih fakulteta za rad s matematički darovitom djecom u dobi od devet do četrnaest godina. Ono je također iskorak u traženju rješenja kako prepoznati takvu djecu, te što i kako s njima raditi, odnosno kako im pomoći u razvoju njihova talenta. Istodobno, ovom se knjigom daje predložak i poticaj za prepoznavanje darovitih u drugim prirodnim područjima, pa čak i šire. Ukratko, ova znanstvena studija se bavi važnim segmentom tematike o kojoj se redovito govorilo na dosadašnjim skupovima *Matematika i dijete*.

Studija ima pet poglavlja: Promjene u kurikulumu *Metodika nastave matematike* na učiteljskim studijima; Razvoj kurikula na učiteljskim studijima za prepoznavanje, izobrazbu i podršku darovitih učenika; Prepoznavanje matematički darovite djece i ekspertni sustav *Mat-dar*; Matematički sadržaji i metode učenja s darovitim učenicima; Neki oblici podrške razvoju matematičkoga dara u učenika osnovne škole.

Nadopunjena je s raznim prilogima: popisi polaznika *Male matematičke škole*, njihovih roditelja, učitelja, škola, te fotografije sa završnih «kviz» natjecanja; popisi školskih psihologa koji su sudjelovali u provedenim istraživanjima; popisi studenata koji su položili izborni kolegij *Matematika i nadareni učenici*; internet adrese tematski vezane uz darovitost i one namjenjene darovitima; uz svako poglavlje navedena je korištena literatura.

Knjiga je rezultat iskustava i promišljanja profesorice Pavleković kako poboljšati sveučilišnu nastavu u segmentu poduke budućih učitelja te kako rad s darovitom djecom općenito, a matematički darovitom posebno, osmisliti i realizirati u našoj sredini, pa i šire, tako da taj rad i pristup budu stručno utemeljeni i opravdani. Potaknuta osobnim razmišljanjima i spoznajama, analiziranjem razine zadovoljstva studenata učiteljskih studija na osječkom sveučilištu, njihovim kompetencijama u području izobrazbe matematički darovitih učenika te uvažavajući promjene nastale primjenom bolonjskog procesa studija, a i rezultate aktualnih istraživanja iz nastave matematike koja su se zadnjih

godina provodila u svijetu, dr.sc. Margita Pavleković se najprije angažirala na promjenama u kolegiju *Metodika nastave matematike* na Učiteljskom fakultetu u Osijeku, posljedice kojih je pažljivo pratila.

Kako bi se dala prilika za punim razvojem matematičkoga dara učenika, valjalo je rast kompetencija studenata učiteljskih studija premjestiti u uvjete izvanškolske izobrazbe darovitih učenika. U tu je svrhu profesorica Pavleković osmislila tzv. ***Malu matematičku školu***, a studentima ponudila izborni kolegij *Matematika i nadareni učenici*. U knjizi je opisan taj originalno zamišljeni oblik edukacije studenata za izobrazbu matematički darovitih učenika. Mala matematička škola odvija se (i danas) u prostorima osječkog Učiteljskog fakulteta, osnovana je podrškom fakulteta te ravnatelja i učitelja osnovnih škola i roditelja učenika četvrtih razreda, dodatno zainteresiranih za matematiku. Kasnije je obuhvatila rad s djecom od četvrtog do osmog razreda. Za kontinuirani rad Male matematičke škole prof. dr. sc. Margita Pavleković zahvaljivala je mnogima na „stalnoj i bezrezervnoj potpori“. Međutim, jasno je da bez njezine zamisli, osmišljavanja sadržaja, plana provođenja i sveukupne organizacije i realizacije, stalnog poboljšavanja i unapređivanja, te škole ne bi bilo. *Mala matematička škola* uvedena je 2003. godine. Od tada, u prostorijama fakulteta studenti završne godine učiteljskih studija, dva sata tjedno tijekom svake školske godine osmišljavaju i provode izvanškolsku nastavu matematike s pedesetak desetgodišnjaka osječkih osnovnih škola koji pokazuju poseban interes za matematiku. Prve dvije godine suradničkoga rada nastavnika i studenata s darovitim učenicima četvrtoga razreda u Maloj matematičkoj školi rezultirale su izbornim kolegijem *Matematika i nadareni učenici* koji je odobren 2005. godine u okviru petogodišnjeg sveučilišnog integriranog preddiplomskog i diplomskog učiteljskog studija. Međutim, da bi se zamišljeno moglo uspješno realizirati bilo je potrebno iznaći načine, metode, postupke za prepoznavanje matematički potencijalno darovite djece. U tu je svrhu profesorica Pavleković kreirala ekspertni sustav *Mat-dar* uz pomoć niza stručnjaka iz drugih područja, a korištenjem *programske podrške Exsys*. Taj se sustav također pokazao učinkovitim metodološkim alatom i u edukaciji studenata, a ne samo kao pomoć učiteljima.

U knjizi se razrađuje i obrazlaže cijeli niz matematičkih tema i raznih oblika rada koji su pogodni za matematički darovitu djecu; važno je što dominira nestandardnost i poticajnost, primjerenost i atraktivnost. Autorica se cijelo vrijeme vodi svješću da je za

prepoznavanje darovite djece i rad s njima, nužno “izgraditi” kompetentne učitelje koji će znati, htjeti i moći to učiniti: takvu djecu prepoznati, s njima raditi i biti im od pomoći i koristi. Sigurno je da bi opisani model edukacije studenata (budućih učitelja) koji je profesorica Margita Pavleković uz pomoć suradnika provela na svojem Učiteljskom fakultetu valjalo prihvatiti i provoditi na drugim našim učiteljskim fakultetima. Knjiga pruža cijeli niz sadržaja koji pomažu u prepoznavanju darovitih i u radu s darovitima, a ponuđene ideje su poticajne i za druge struke, ne samo za matematiku.

Dokazala se ispravnost posvećenosti dr.sc. Margite Pavleković problematici prepoznavanja darovitih te otkrivanju i nalaženju aktivnosti koje će pomoći u daljnjem razvoju i realizaciji dječjega dara za matematiku. A komparativne analize i empirijski rezultati istraživanja ponovo su potvrdili bjelodanu činjenicu da dobra i kvalitetna suradnja škole, fakulteta i roditelja koristi svim sudionicima obrazovnoga procesa i sustava u društvu. Na str. 124. stoji: “U svijetu je već dugo prepoznata važnost pozitivnog pristupa matematici i mnogo se daje na popularizaciju matematike. Kod nas je popularniji pristup matematici i znanosti općenito još uvijek u poveljama. Polaznici *Male matematičke škole* postali su popularizatori matematike u svojoj sredini, među svojim vršnjacima. Studenti prve generacije iz 2003./04. godine dugo su s osmijehom komentirali izjavu polaznika koji svojem vršnjaku objašnjava ”ja svima pričam da idem na fakultet učiti matematiku, a nitko mi ne vjeruje”.

Cijeli radni vijek profesorice Pavleković je bio u ozračju matematičke edukacije što je i rezultiralo dvjema knjigama, sveučilišnim udžbenicima ***Metodika nastave matematike s informatikom I, II***. Prva je objavljena 1997. i ponovljeno tiskana 2001. i 2008., a druga je izašla 1999., obje kod izdavačke kuće “Element”. Knjige su nastale kao posljedica višegodišnjeg uspješnog predavanja kolegija istih naslova studentima treće i četvrte godine studija matematike i fizike te matematike i informatike, ali, valja naglasiti, i dugogodišnje suradnje s profesorom Borisom Pavkovićem. U predgovoru prve knjige stoji: “I na kraju, ali prije svega, zahvaljujem prof. dr. B. Pavkoviću koji mi je prije desetak godina ustupio svoja predavanja i ostao stalna podrška u svim mojim traganjima za novinama u realizaciji ovog značajnog kolegija struke. U ime mnogih koje je prof. dr. B. Pavković poučio Metodici matematike još jednom hvala.” A u uvodu druge knjige piše: “... zahvaljujem svojim recenzentima, u prvom redu prof. dr. B. Pavkoviću na nesebičnoj podršci, u dugom

nizu godina, koju sam imala pri upućivanju studenata matematike u izuzetno lijep, ali ni malo lak poziv učitelja matematike.” Pojava ovih sveučilišnih udžbenika unaprijedila je nastavu matematike i informatike na našim pedagoškim fakultetima, ali i znatno šire, pa posljedično i nastavu matematike u našim školama.

Završno poglavlje druge knjige govori *O proturječnim zahtjevima na izvođenje nastavnog sata*, otvoreno i jasno raspravlja o zahtjevima koji su postavljeni na nastavni proces, a koji su često suprotstavljeni te su stoga izvor mnogih poteškoća u realizaciji nastave, izvor mnogih frustracija i učenika i učitelja.

Tako je bilo u vrijeme pisanja ovih knjiga, tako je i danas. Sa svake stranice ovih knjiga profesorice Pavleković izbija njezina ljubav prema poučavanju i nastojanje da se bez ostatka ta ljubav prenese na čitatelje, buduće i sadašnje učitelje. Druga je konstanta njezin početni i nezaobilazni, stalni zahtjev da nastava u svakom trenutku, bez izuzetka, mora biti do kraja podređena učeniku, mora uvažavati njegovu osobnost, njegove sposobnosti i kvalitete.

“Vječno razapet proturječnim zahtjevima u svezi s metodama i oblicima rada, opsegom i dubinom matematičkih sadržaja, izložen sučeljavanju tradicionalnog i suvremenog u matematici, s uvijek novim licima – učitelj, u skladu sa svojim znanjima i vještinama, osmišljava ozračje aktivnog učenja na satu matematike.

Stiješnjen vremenom i razdiran željom što više toga otkriti sa svojim učenicima lomit će se između verbalne, predavačke metode i individualiziranog rada. (...) Morat će naći mjeru između nastojanja da svaki učenik napreduje prema svojim sposobnostima i svoje želje da se što više posveti najboljima. Morat će se, ocjenjujući svoje učenike, naviknuti na vječno propitivanje svoje savjesti: jesam li bio dosljedan, temeljit, jesam li podcijenio ili precijenio znanje nekog učenika, jesam li ga pritom povrijedio, jesam li možda podržao njegovu lijenost? (...) Svaki je učenik poštovanja vrijedna osoba.

Kada se za dvadesetak godina učitelj sretne s učenikom, njihove uloge u društvu bit će znatno izmijenjene. (Ne)ugoda pri susretu učitelja i učenika puno godina poslije poučavanja, govorit će koliko je ta veza bila plodonosna u vremenu poučavanja.” (Metodika II, str. 185).

Sve ovdje napisano i navedeno o sveučilišnim udžbenicima *Metodika nastave matematike s informatikom I, II*, knjizi *Matematika i nadareni učenici* i projektu *Mala*

*matematička škola*, pokazuje i dokazuje kako je svojedobno pokretanje međunarodnog znanstvenog skupa *Matematika i dijete* posljedica dugogodišnjih promišljanja, skupljenih iskustava, teoretskog i praktičnog rada te jasnih i zrelih stavova profesorice dr.sc. Margite Pavleković. I mada je njezin odlazak nenadomjestiv, skup je potpuno jasno koncipiran u svakom pogledu, pa će moći i dalje trajati na dobrobit matematike i onih koji je poučavaju i koje se poučava.

Vjerujemo da će ovaj kolokvij u buduću biti s dopunjenim/proširenim nazivom *Međunarodni znanstveni skup "Margita Pavleković" - Matematika i dijete*.

Mirko Polonijo



# ABSTRACTS

## **Understanding of mathematically gifted students' approaches to problem solving**

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*Abstract.* Problem solving in mathematics is a leading creative activity and it must be offered (if not to everyone) at least to mathematically gifted students at all levels of education. The research has shown that mathematically gifted students themselves recognize mathematical problem solving to be the most important component for developing their potentials in mathematics. When researching problem-solving activities in relation to students' success one could focus on various issues: mental schemes, generalising, procedural and conceptual problems, heuristics . . . In this paper we focus on analysing how mathematically gifted students (from 6th to 9th grade of elementary school, age 12 to 15) solved a particular problem (we named it 'the prisoner cell problem') where the skills of generalising and establishing some rules are needed for the solution. We investigate students' solutions to the problem from the perspective of the strategies used, correctness of the solutions at different extensions of the problem in relation to the strategies used, and from the point of relation between their age and success in problem solving. Such an analysis gives us better understanding of the development of mental schemes among mathematically gifted students for solving problems. The research contributes to more systematic and organised work with gifted students in the mathematics classroom. We also want to encourage teachers of mathematics to integrate problem solving into their teaching, since this would be challenging for gifted students and it would enable them to fulfil and develop their potentials. We demonstrate some students' problem-solving solutions and their approaches in order to give mathematics teachers some practical examples which could be starting points both for discussion with students and for challenging them as they learn mathematics.

*Keywords:* mathematically gifted student, problem solving, generalisation, mental scheme, problem-solving strategies

## **Contemporary methods of teaching mathematics – the discovering algorithm method. Algorithm for fraction division**

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*Abstract.* The requirements of contemporary life imposed by rapid economic growth and development in the world, and in Croatia, impose new goals and requirements on education. New requirements highlight new values and competences that have not been in focus until recently but have been characterized as competences owned only by individuals. However, flexible problem-solving skills and establishing connections among new information, as well as logical thinking and linking all aspects of knowledge, is no longer a characteristic needed only by single people, but the general population in a world of rapid technological changes and economic development. Such a combination of skills and competence of individuals should be developed through the education system. Mathematics as a subject, by its nature and content is favorable in achieving such goals. Yet the traditional methods of teaching mathematics focused on the adoption of procedural knowledge can only develop such skills with the individual. However, the introduction of new methods of teaching mathematics can enhance traditional teaching and provide the development of necessary skills to a wider population of pupils.

This paper describes one of the modern methods of teaching mathematics, as well as the impact that this method has on procedural knowledge and on the development of conceptual knowledge. Research on the impact of the discovering algorithm method for fraction division in procedural and conceptual knowledge of fraction dividing was conducted on a sample of 241 pupils of the sixth grade of elementary school, divided into test and control groups. The results showed a positive influence of the discovering algorithm method for fraction division on procedural and conceptual knowledge in relation to the traditional method of demonstration procedures and training.

*Keywords:* algorithm, division of fractions, conceptual knowledge, procedural knowledge

## **Word problems in mathematics teaching**

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*Abstract.* General problem-solving skills are of central importance in school mathematics achievement. Word problems play an important role not just in mathematical education, but in general education as well. Word problem solving, as well as comprehension and text interpretation are present among the skills in the mathematical competence model. Word problems also play a crucial role in forming the concept of operations and indirectly in practicing operations. Working with word problems in elementary education creates a base for the ability to model more complex, practical problems. Word problems also help in developing comprehension, judging, memorization and self-check abilities.

In order to efficiently enhance students' problem solving skills they should be assigned word problems which are new to them and to which they themselves have to find the steps to the solution, the algorithm.

A number of researches, experiments and scientific papers in the didactics of mathematics prove that the role of visual representations in word problem solving is essential. Visual representation often helps in understanding a problem. Using visual representations leads to a better understanding and to improving special mathematical reasoning.

In all stages of education we must place great emphasis on word problems, on their correct interpretation, understanding, on observing the steps in problem solving, possible representations, interpreting results in terms of real world situations because word problems play an important role in developing comprehension.

The aim of this research was to measure students' word problem solving skills, as well as to investigate the way they can actively apply their knowledge when solving problems directly and not directly connected to the curriculum. We have investigated the relationship between different knowledge areas and levels in the case

of Primary School and Kindergarten Teacher Training College's students at Partium Christian University Oradea.

*Keywords:* teacher training, word problems, reading, arithmetical problem-solving methods, primary school textbooks

## **Graphical representations in teaching GCF and LCM**

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*Abstract.* The greatest common factor (GCF) and the least common multiple (LCM) of numbers are concepts which have been shown in literature to be calculated by students in rule-based manner. Therefore, students' knowledge is procedural and students usually cannot provide intuitive or logical explanation for procedure. Common method for finding the GCF and the LCM employ factoring. This method is based on symbolical representation of concepts of GCF, LCM and factorization.

In this paper, we describe alternative approach for finding GCF and LCM. This approach is based on graphical representation of concepts of LCF and LCM and includes three models: Venn diagrams, line method and area method for finding GCF and LCM. Taking into account role of visualization, we propose the use of graphical models as teaching tools in order to reinforce students' understanding of GCF and LCM.

*Keywords:* greatest common factor, least common multiple, graphical representation, factorization, teaching tools

## **Mathematics + Computer Science = True**

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*Abstract.* Mathematics is a fundamental tool for many sciences. Even so, they are often taught as completely separate topics in higher education. Sometimes math problems are taken from sciences like physics or from real life examples and even if some parts of the mathematics needed is briefly explained when teaching sciences, it is often assumed that the knowledge and understanding of mathematics has already been acquired to a sufficient level before the course starts. However, it is easy to see that a deeper understanding of the mathematics behind the problem in question, also helps in acquiring a better and most of all a deeper understanding of the problem itself. So, the question therefore is how this can be done in computer science in higher education? In other words, how can computer science benefit from including some teaching of mathematics and vice versa? Examples from a mixed course in mathematics and computer graphics will be given and experiences from teaching graphics and mathematics will be discussed. Moreover, experiences from other levels of education will be given and also an overview of how Uppsala University in Sweden supports teachers with a variety of pedagogical and didactic courses as well as other initiatives.

*Keywords:* teaching, mathematics, computer graphics, higher education

## **Discovering patterns of student behaviour in e-learning environment**

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*Abstract.* The benefits of e-learning have been widely recognized in today's education. However, behaviour of students attending a course through an e-learning platform and its connection to students' satisfaction with a course is still not investigated enough. This paper analyses a course log data in e-learning environment in addition to some students' descriptive variables at University of Osijek, and aims to discover patterns in students' behaviour that could enable to create profiles of satisfied and unsatisfied students. The final purpose is to reveal knowledge about student behaviour that will assist academic teachers in increasing the level of their students' satisfaction. The methodology used in the research includes several data mining methods, such as statistical tests of dependence, and support vector machines. The results show that satisfied students put more effort in frequent viewing of all course materials, they are more active in uploading assignments, and have more previously earned ECTS points than unsatisfied students. The extracted characteristics could be used to improve student satisfaction with the course by stimulating those activities. In order to generalize results, the research is to be extended to include more e-courses on different levels of academic education.

*Keywords:* student behaviour, e-learning, data mining, classification trees, support vector machines

## **Classification trees in detecting students' motivation for maths from their ICT and Facebook use**

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*Abstract.* Previous studies show that students' motivation for maths is an important factor of students' active engagement in the learning process and the level of students' achievement in mathematics courses. The purpose of this paper is to present the potential of classification trees in detecting students' motivation for maths. In addition, the aim was to construct a classification tree model that will be effective in detecting students with a lower level of motivation for maths. The classification tree model was established on students' perception of their ICT and Facebook use. Motivated Strategies for Learning Questionnaire (MSLQ) (Pintrich et al., 1991) scale was used for determining the level of students' motivation for maths, and the Facebook Intensity (FI) scale (Ellison et al., 2007) was used for measuring students' Facebook use. The students from the third year of the Faculty of Education in Osijek participated in this research. The results showed great potential of a classification tree model as means for detecting students' motivation for maths since the best obtained classification tree model achieved accuracy of 80% in detecting students with a lower level of motivation for maths.

*Keywords:* classification tree, motivation for maths, ICT usage, Facebook usage, students' attitude

## **Using Moodle in teaching mathematics in Croatian education system**

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*Abstract.* Moodle (**M**odular **O**bject-**O**riented **D**ynamic **L**earning **E**nvironment) is an open-source computer system for creating online courses (Macmillan Dictionary). This is a Learning Management System for online or hybrid teaching and learning. In this article it will be presented how Croatian educators use Moodle in teaching mathematics through examples from successful practice at all levels of education.

The purpose of this article is to show a large range of options offered by Moodle to make teaching of mathematics innovating, more interactive and interesting; to find out the most commonly used tools in math courses and to present the results obtained from a questionnaire conducted among teachers who have Moodle courses. The author also analyses different type of performance of courses, reasons for using a certain course and both students' and teachers' satisfaction with the courses.

*Keywords:* Moodle, teaching mathematics, education, distance learning, on-line quiz

## **Future teachers' perception on the application of ICT in the process of assessment and feedback**

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*Abstract.* In recent years the quality of education has been the main focus in higher education. Feedback is a key element of formative assessment or even a key part of the overall approach to learning. Information and communication technology enables more effective ways of teaching with frequent formative assessment and timely feedback. The aim of this study is to examine the extent to which the future teachers are familiar with the concepts of summative and formative assessment, and their perception of the role of ICT in these processes, and to what extent, in this context, formative assessment and feedback appear in their current education. Results showed that future teachers are not very well familiar with these concepts, but are mostly satisfied with communication through ICT although in spite of the emphasized deficiencies. Limitations and implications for further research are considered.

*Keywords:* feedback, formative assessment, summative assessment, information and communication technologies, future teachers.

## **Lotfi A. Zadeh: one man STEM**

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*Abstract.* Lotfi A. Zadeh is a well-known electrical engineer, computer scientist, and mathematician. Honouring his 94<sup>th</sup> birthday this paper will evaluate and analyze his work and influence on modern science, practice and philosophy in STEM area. He is primarily known as the founder of fuzzy logic, and for his work on z-transforms as discrete equivalent of continuous transforms as well. Just the fact that he has received 23 honorary PhDs contributes to his identity of a recognized and appreciated scientist. Criticism of fuzzy logic will also be presented in this paper. The list of his references and heritage regarding STEM analysis is long. Personally speaking, I can say that this man and his work have, and still do, deeply influence my career.

*Keywords:* Lotfi A. Zadeh, fuzzy logic, computing with words, heritage, STEM

## **Pass rates in mathematical courses: relationship with the state matura exams scores and high school grades**

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*Abstract.* In this paper authors investigate relationship between scores in state Matura exams in mathematics, Croatian language, high school grades and success in some mathematical courses in the undergraduate study of “Information and Business Systems” at the University of Zagreb, Faculty of Organization and Informatics. Mathematical courses are often courses with the lowest pass rates and the lowest average grade. Therefore, identification of required knowledge for successful passage of mathematical courses influences success rate for the whole study. Methods used in the paper are primary statistical and data mining methods: descriptive statistics, logistic regression and others. Computation is done in the R programming language.

*Keywords:* study success rate, math-courses success rate, state Matura exam results, data mining methods, R programming

## **Approaches to teaching mathematics in lower primary education**

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*Abstract.* This paper aims at establishing if primary school teachers in our schools apply innovative approaches in teaching mathematics to students in grades 1 to 5, with an emphasis on an integrated approach to teaching. A traditional approach to teaching still prevails in our educational system, especially in teaching mathematics. Modernization of the teaching process and introduction of innovative models of work into teaching mathematics is the primary task of educational system, but also of every individual teacher. Schools needs to modernize, to use new, innovative models and ways of work, so that classes can become more efficient, creative and rational and that students can gain functional knowledge and abilities.

The first part of the paper provides explanations of basic theoretical terms related to this research topic, the second part introduces research methodology for the specified issue, while the third part presents the analysis and interpretation of the obtained results.

*Keywords:* teaching, mathematics, teacher, student, approaches to teaching, innovative models of teaching

## **Issues in contemporary teaching of mathematics and teacher competencies**

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*Abstract.* In this paper, the author examines current problems in the teaching of mathematics and mathematics education and shows how theoretical and practical findings underestimate the role of the social dimension in the teaching of mathematics. Mathematical education is perceived in this research as a kind of a social construct in contrast to the traditional definition of (teaching) mathematics as a purely scientific discipline. The author focuses on students' emotional reactions, classroom environment and teacher competencies as indicators of quality in the teaching of mathematics. In the context of contemporary mathematics teaching, the author uses a critical approach to assess the way and content of teacher education as well as the required competencies in quality mathematics teachers. Taking into account the requirements and specifics of teaching mathematics that are in the domain of pedagogical theory and practice, the author emphasises teachers' pedagogical competencies and their definition from the pedagogical perspective. A review of relevant research has led the author to state that a stimulating classroom environment is one of the key assumptions of students' success in mathematics and that the didactic-methodological guidelines of teaching mathematics should be based on the individuality of students and the demands and difficulties they encounter in the classroom.

*Keywords:* teaching of mathematics, students' emotional reactions, classroom environment, mathematics teachers' pedagogical competencies

## **Teaching Mathematics in early education: current issues in classrooms**

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*Abstract.* Social perception of teaching mathematics in early education is often formally utilized, i.e. it is interpreted as irrelevant and inappropriate for children aged three to seven. In order to find out if mathematics is present in early education, a survey was conducted in 37 kindergartens in three Croatian counties and one canton in Bosnia and Herzegovina. For that purpose *Scale for assessing surroundings and interaction* was constructed (scale's Cronbach's alpha .93), and its subscale *Mathematics* was applied (subscales' Cronbach's alpha .78). The results show that the frequency of mathematical activities in classrooms correlates with preschool teachers' levels of formal education, chronological structure of a classroom, and overall number of children in a classroom. So, three-year-olds and mixed age groups have fewer opportunities for mathematical activities, while six-year-olds have mathematical activities on a daily basis. Also, in the groups with less than 15 children, and groups with more than 25 children, math activities are occasional. As far as the structure of materials and type of activities are concerned, preschool teachers organize such activities that allow younger kindergarten children to practice comparing quantity and recognizing patterns (concrete objects, mostly building blocks and jigsaw puzzles) while older kindergarten children can enjoy counting and measuring materials of diverse structures, and geometry. The results suggest that preschool teachers implicitly see mathematics as an academic activity, and therefore design it accordingly to the concept of school readiness. On a pragmatic level, results can be used as a turning point for math activities in the context of early education, i.e. an argument for questioning contemporary pedagogical practice regarding teaching mathematics, and scrutinizing the role of preschool teachers in that area.

*Keywords:* early education, math activities, preschool teachers, preschool children

*G. R. (boy, 5 yrs.) throwing stones in the water:*

*“Look at this stone. When it drops into water,  
it gives its energy to the water. . . ”*

## **Preservice mathematics teachers' problem solving processes when working on two nonroutine geometry problems**

Doris Dumičić Danilović and Sanja Rukavina

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*Abstract.* Educational researchers widely recognize the importance of the development of students' metacognitive strategies and problem solving skills in order to improve their mathematics achievement. However, the time dedicated to the study and development of preservice mathematics teachers' problem solving skills is minimal. The aim of this research is to provide the insight into the metacognitive behaviour of six preservice mathematics teachers, attending graduate teacher training courses at the Department of Mathematics of the University of Rijeka, while solving nonroutine geometry problems.

This study was designed as a multiple case study, a qualitative research method was employed. We observed some weaknesses in students' mathematical knowledge with particular focus on how they relate and transfer their conceptual and procedural knowledge to unfamiliar problem situations. Our intention was to explore problem solving process experienced by preservice teachers faced with nonroutine geometry problems, working individually within dynamic geometry environment (GeoGebra) or paper-pencil environment, and how the use of dynamic geometry software can influence on participant's decision-making, reflections and problem solving process.

*Keywords:* Problem solving process, Metacognition, Preservice mathematics teacher, Nonroutine geometry problem, Dynamic geometry environment

## **Tendencies in identifying geometric shapes observed in photos of real objects – case of students of primary education**

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*Abstract.* One of the tasks of initial teaching of geometry is developing students' spatial reasoning. Association of a geometrical content with real situations and objects is recommended for the realization of this task. Therefore, competencies related to spatial thinking, precisely spatial visualization, should be found among the competencies of the teachers. Among teachers' spatial visualization abilities, we can point out the ability to recognize geometric shapes met in different environments, and the ability to accurately and precisely describe these shapes using geometric terminology.

In this paper, we analyze (quantitatively and qualitatively) responses of 85 students of primary education in activity of identifying geometric shapes at 8 photos of real objects.

The analysis of students' answers is focused on the following objectives:

- to analyze the terms listed in the students' responses with special emphasis on accuracy, frequency and diversity of responses;
- to analyze and compare the use of plane geometry and solid geometry terms in students' answers;
- to classify students' incorrect answers.

Results indicate that each student stated 29 terms in average: 25 correct terms and 4 incorrect terms. There are eight different terms among the most frequent correct answers. The tendency in indicating plane figures more frequently than solids can be observed. More than two thirds of answers indicate 2D figures: 69.8% among all given answers and 70.9% of total correct answers. Even the results are satisfactory, the results point to the need for introducing tasks of this type in the training of future teachers.

*Keywords:* spatial visualization, geometric shape, plane figure, solid, identification

## **Visual mathematics and geometry, the “final” step: projective geometry through linear algebrae**

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*Abstract.* The renaissance painters, *Leonardo da Vinci* and *Albrecht Dürer* analysed first the viewing process in the so-called *practical perspective*, and introduced the concept of the *point at infinity* as *ideal common point of parallel lines*. Namely, such ideal points can have proper image points in the *horizontal line*. This process led 300-400 years later to the *projective geometry* as a base of different non-Euclidean geometries. At the same time the free-hand-drawing and painting got their scientific base. Then *optics* developed with fantastic tools as glasses, telescopes, microscopes, etc.

After a more popular introduction to the classical projective space, we will illustrate our topic more sketchily by figures. Mathematics helped this process by introducing *homogeneous coordinates* and so by  $d + 1$ -dimensional vector spaces for  $d$ -dimensional geometries. On this basis nowadays we can design complicated and attractive pictures, animate moving, make fantastic films by computer.

The first author gratefully remembers to his teacher of drawing *Dezső Horváth* who introduced him into this knowledge in a primary school in Győr. The third author's math teacher in this topic was *Zoltán Dunay*, even a student of the first author. The influence of a charismatic teacher is always important and decisive in the development of an interested student.

*Keywords:* Euclidean-projective visualization, projective plane and space by linear algebra

**Mathematics Subject Classification 2010:** 15A75, 51N15, 65D18, 68U10.

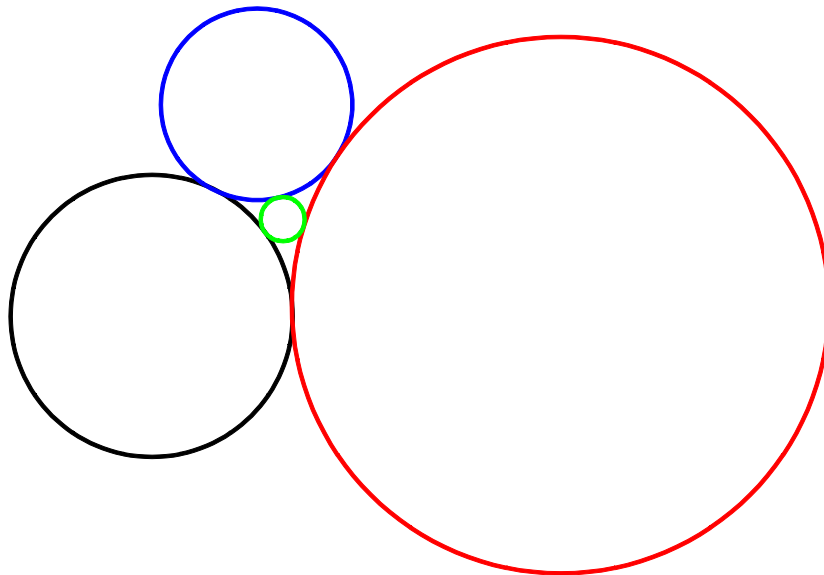
## **Is any angle a right angle?**

Vladimir Volenec

Department of Mathematics, Faculty of Science, University of Zagreb, Croatia

*Abstract.* The explanation of the ostensible paradox, where it is shown that any angle is a right angle, is done by means of the well-known Descartes-Soddy's formula for the curvature of the four circles, which touch each other.

*Keywords:* triangle, circle, curvature, Soddy's formula



## An interesting analogy of Kimberling-Yff's problem

Zdenka Kolar-Begović<sup>1</sup>, Ružica Kolar-Šuper<sup>2</sup> and Vladimir Volenec<sup>3</sup>

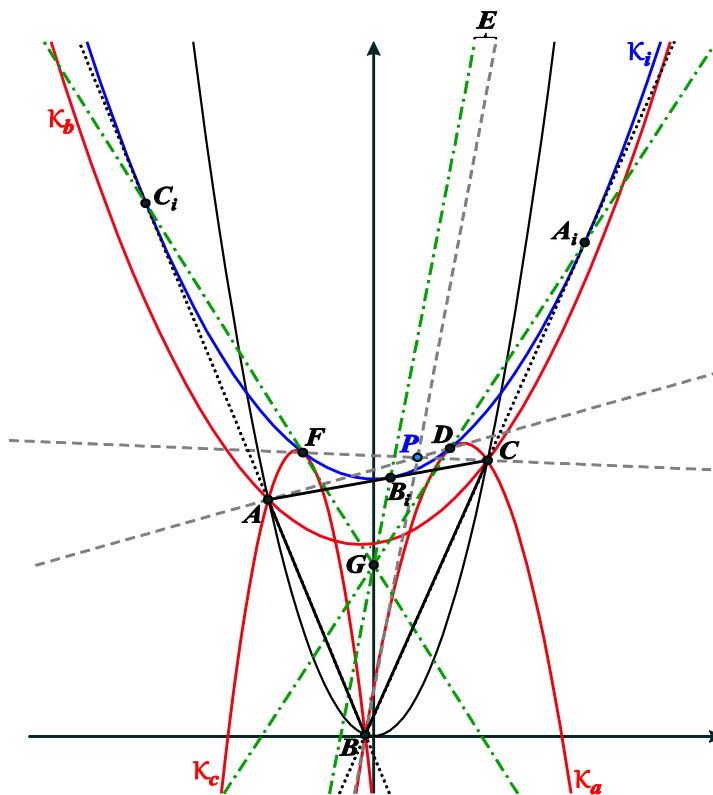
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*Abstract.* The existence of three circles touching the inscribed circle of an allowable triangle in an isotropic plane and going through two vertices of a considered triangle is proved in this paper. Some relations between these three circles of a triangle and elements of a triangle are investigated. Formulae for their radii are also given.

*Keywords:* isotropic plane, standard triangle, Kimberling-Yff's circles



## **Pre-service teachers and statistics: an empirical study about attitudes and reasoning**

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*Abstract.* In this paper, we will discuss pre-service elementary teachers' attitudes towards statistics, as we believe that these attitudes have a key role in the teaching and learning process. The attitudes were examined using the Scale of Attitudes Towards Statistics and the analysis of results showed pre-service elementary teachers' neutral attitude toward statistics. Also, we examined their statistical reasoning in measures of center, using several items from Quantitative Reasoning Quotient Test. This analysis showed that pre-service teachers' statistical reasoning is inconsistent from item to item, or topic to topic, depending on the context of the problem. Results of both assessment instruments indicate that pre-service teachers lack awareness of the usefulness of statistics in everyday life and lack of experience of solving everyday problems. Since the attitudes have a significant effect on the learning process, we believe that not knowing where, why and when statistics can or will be used, influenced on pre-service elementary teachers' statistical reasoning.

*Keywords:* attitudes, reasoning, pre-service teachers, statistics, measures of center

**Beliefs about mathematics and mathematics teaching  
of students in mathematics education programme  
at the Department of Mathematics, University of Zagreb**

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*Abstract.* TEDS-M (Teacher Education and Development Study in Mathematics) is an international comparative study of mathematics teacher education. It is concerned with the context, structure, and quality assurance of mathematics teacher education and was recently conducted in 17 countries. Motivated by this study and in order to compare the programme of mathematics teacher education at the Department of Mathematics, University of Zagreb, with the findings obtained in TEDS-M study, we have conducted a survey based on the (adapted) instrument of the TEDS-M. The survey was carried out in 2014 on the population of the fifth year mathematics education students. We have addressed the following two main topics: beliefs about mathematics and mathematics teaching, and opportunities to learn during the teacher education program. The questionnaire contained questions about students' beliefs on nature of mathematics, learning of mathematics, mathematics achievement, preparedness for teaching mathematics, program effectiveness and coherence, as well as on opportunities to learn school and university-level mathematics, mathematics didactics, general education, pedagogy and how to teach, and to gain school experience and field practice.

In this communication, we present our view on the actual mathematics teacher education program at the Department, some findings of our survey and their comparison with the results of the TEDS-M participating countries.

*Keywords:* mathematics teacher education, mathematical and pedagogical knowledge relevant to teaching, beliefs and perspectives on content and pedagogy, opportunities to learn, TEDS-M

## **Self-reported creativity of primary school teachers and students of teacher studies in diverse domains, and implications of creativity relationships to teaching mathematics in the primary school**

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*Abstract.* Primary school teachers' and students' of teacher studies self-reported creativity in diverse domains and relationships of domain-specific creativity to self-ratings of the creativity in teaching mathematics, language, physical education, visual arts, music, and science, were explored in this study. Teachers in primary schools ( $N = 105$ ) and students of teacher studies ( $N = 111$ ) were the study participants (sample age ranged 21–64 years;  $M = 31.2$ ,  $SD = 11.6$ ; 95.4% women). Sixty-two lesson plans for teaching mathematics in grades 1–4 written by students of teacher studies, were analyzed regarding their contents (i.e. chosen activities that map onto different creativity domains). The results show that when the students of teacher studies rated themselves as generally creative, they also rated themselves as creative to a different degree, for example, the area that had the lowest and not significant correlation with general creativity ratings was mathematics. This internally experienced subjective structure of creativity in students, as well as the meaning of being creative (excluding mathematics), may have implications in teaching mathematics in primary school, such as the choice of activities during mathematics lessons. Age- and expertise-related differences between teachers and students of teacher studies were found as well, bringing up numerous questions on (creative) teaching of mathematics and its relationship to teaching excellence and student outcomes.

*Keywords:* mathematics teaching, creativity domains, implicit theories, lesson preparations, art bias

## **How Croatian mathematics teachers organize their teaching in lower secondary classrooms: differences according to the initial education**

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*Abstract.* Initial education of mathematics teachers is important for teaching and learning mathematics in school classrooms. Some studies showed that there exists a relation between students' achievement and teachers' mathematical and pedagogical content knowledge. In Croatian lower secondary education, there are mathematics teachers who significantly differ in their initial education. In this paper, we examine teaching practices of two groups of mathematics teachers; those who finished former pedagogical academies and those who obtained their degrees from departments of mathematics. Using qualitative methods as observations and interviews, we investigated the teaching practice of 12 lower secondary mathematics teachers with special reference to the utilization of the textbook. Results showed that these two groups of teachers differ in the use of textbooks, but also in some other parts of teaching practice.

*Keywords:* mathematics teacher, initial education, textbook, teaching practice, qualitative study

## **Structures of Croatian mathematics textbooks**

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*Abstract.* Textbooks can be seen as a teaching tool with a wide potential use in school, but also as the intermediaries between the planned and implemented curriculum. They greatly affect the way in which the intended mathematical content is transformed into actions and educational opportunities inside classrooms. As such they are conceptualized as part of the potentially implemented curriculum in the TIMSS curriculum framework. Thus it is important to analyze the structure of textbooks in order to see the potential pedagogical implications.

We use TIMSS mathematics framework to study structure of textbooks for the last grade of secondary school in Croatia (Population 3 in TIMSS analysis). Also we try to investigate variability of these textbooks inside the timeframe of the past 20 years.

*Keywords:* TIMSS, textbooks, structure, mathematics, secondary school

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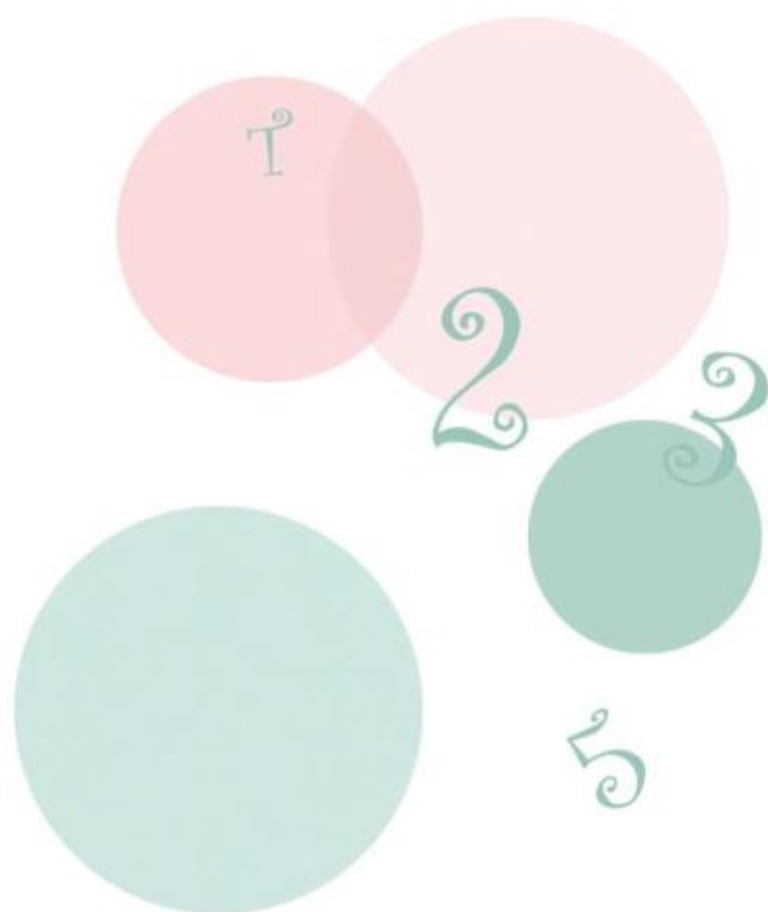
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