

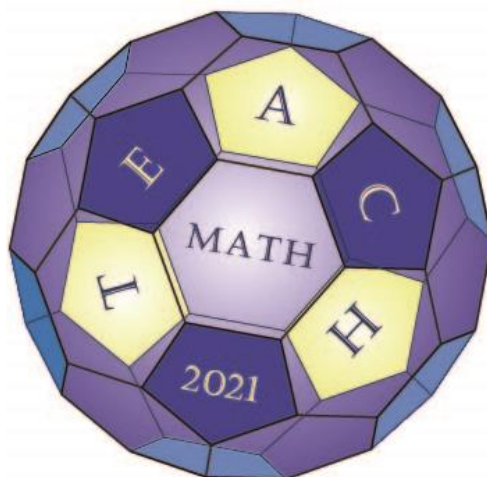
Josip Juraj Strossmayer University of Osijek



Faculty of Education



Department of Mathematics



**The 8th International Scientific Colloquium
MATHEMATICS AND CHILDREN
founded by Margita Pavleković**

**PROGRAM
&
BOOK OF ABSTRACTS**

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**Zdenka Kolar-Begović
Ružica Kolar-Šuper
Ana Katalenić**

Croatia, Osijek, May 28-29, 2021

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**Josip Juraj Strossmayer University of Osijek
Faculty of Education and Department of Mathematics**

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PROGRAM

FRIDAY, May 28

9.00 - 9.30 **OPENING**

Morning Session

9.30-10.15	Plenary lecture Vida Manfreda Kolar <i>An analysis of students' misconceptions as a method of improving the teaching and learning of mathematics through better comprehension: The case of fractions</i>
10.15-10.35	Tatjana Hodnik, Janez Krek <i>Beyond dichotomies in mathematics teaching</i>
10.35-10.55	Attila Bölcskei <i>Spatial geometry in University entrance examinations in Hungary 1980-85 vs. 2015-2020</i>
10.55-11.15	Jelena Pleština, Željka Milin Šipuš <i>The transition of the concept of a polynomial through undergraduate mathematics courses</i>
11.15-11.45	Break

11.45-12.05	Maja Cindrić, Lucija Karega <i>Fluency of mental computation of a prospective teachers</i>
12.05-12.25	Dubravka Glasnović Gracin, Ana Kuzle <i>Designated geometry curriculum in Croatian primary education: A critical view</i>
12.25-12.45	Željka Milin Šipuš, Matija Bašić, Eva Špalj, Renata Vlahović Kruc <i>Is it mathematics? Available modelling skills of prospective mathematics teachers</i>
12.45-13.05	Željka Zorić <i>What do mathematics teachers believe about professional development, educational research, and teaching?</i>
13.05-13.25	Kerstin Bräuning, Lena Hertel <i>Potential of children math lesson for investigating peer interactions</i>
13.25-14.30	Break

Afternoon Session

14.30-14.50	Goran Trupčević, Anđa Valent <i>"Multiplication table, at last!" - Teaching of multiplication in Croatia</i>
14.50-15.10	Jasmina Milinković <i>"Concurrent" method of teaching multiplication and division</i>
15.10-15.30	Azra Stašćik, Anita Čorak <i>Open vs. closed tasks in mathematics</i>
15.30-15.50	Marija Gelešić, Ana Katalenić <i>Number representation in textbooks for primary mathematics education in Croatia</i>
15.50-16.10	Edith Debrenti <i>Visualization in the teaching and learning of mathematics</i>
16.10-16.40	Break

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17.00-17.20	Josipa Matotek <i>Attitudes towards online learning among students of Faculty of Civil Engineering and Architecture Osijek</i>
17.20-17.40	Slavica Brkić, Ljiljanka Kvesić, Marina Zubac <i>The role of mathematics in the successful acquisition of teaching material in physics</i>
17.40-18.00	Saša Duka <i>Transformation of population density data in the Republic of Croatia</i>

SATURDAY, May 29

Morning Session

9.00-9.45	Plenary lecture Franka Miriam Brueckler <i>A football trip through mathematics</i>
9.45-10.05	Željka Zorić, Gordan Radobolja, Diana Dumančić, Lucija Miloš <i>Mathduel, a math game for high school</i>
10.05-10.25	Ljerka Jukić Matić, Mirela Jukić Bokun, Darija Marković, Dragana Jankov Maširević, Mihaela Ribičić Penava, Mateja Đumić <i>Exploring the factors that influence the utilization of digital games in the mathematics classroom</i>
10.25-10.45	Marina Furkes <i>Mathematics teachers as popularizers of science</i>
10.45-11.15	Coffee Break

11.15-11.35	Karolina Dobi Barišić, Diana Moslavac Bičvić <i>eTwinning as a potential tool in teacher education</i>
11.35-11.55	Aliya Bukusheva <i>Interdisciplinarity in mathematics teacher education</i>
11.55-12.15	Amanda Glavaš, Azra Staščik <i>Essay as technique of assessing the element of mathematical communication</i>
12.15-12.35	Jelena Kišosondi, Ružica Kolar-Šuper, Ana Katalenić, Zdenka Kolar-Begović <i>Didactic manipulatives in primary mathematics education in Croatia</i>
12.35-12.55	Nives Baranović, Branka Antunović-Piton <i>Ways of discovering general rules of growing geometric patterns by students aged 15 to 17</i>

13.00

CLOSING

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A football trip through mathematics

Franka Miriam Brueckler

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Abstract. Football (or soccer) is as much popular as mathematics is not, at least in the general public and among kids in particular. However, one cannot be a football fan without using some mathematics, as will be demonstrated by examples ranging from 1st to 8th year of primary school. For each grade, we shall give examples of incorporating football topics (of real relevance to anyone interested in football) into the Croatian mathematical curriculum. This ranges from basic arithmetics needed to follow the results of a championship up to the application of the Pythagorean theorem and basic probability to explain penalty statistics.

Keywords: primary school mathematics, football, arithmetics, geometry, probability, algebra

Nogometom kroz matematiku

Franka Miriam Brueckler

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Sažetak. Koliko je nogomet popularan, toliko matematika to nije, bar u široj populaciji i posebno među djecom. No, nemoguće je biti ljubitelj nogometa bez korištenja bar nešto matematike, što ćemo pokazati kroz primjere osnovnoškolske matematike od 1. do 8. razreda. Za svaki razred dat ćemo primjere kako uključiti nogometne teme (koje su stvarno relevantne za svakoga koga zanima nogomet) u hrvatski osnovnoškolski matematički kurikulum. Tako će biti predstavljeni primjeri od osnovne aritmetike potrebne za praćenje rezultata prvenstava do primjena Pitagorinog poučka i osnova vjerojatnosti za objašnjenje statistika kaznenih udaraca.

Ključne riječi: osnovnoškolska matematika, nogomet, aritmetika, geometrija, vjerojatnost, algebra

An analysis of students' misconceptions as a method of improving the teaching and learning of mathematics through better comprehension: The case of fractions

Vida Manfreda Kolar

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Abstract. Students' misconceptions in mathematics can be addressed at various levels, one of which is recognizing errors at a deep, discursive level, which requires a diagnosis of the reasons for observed errors and the involvement of students in dialogue, explanation, and justification of sounder mathematical reasoning. Four typical errors types identified in the diagnosis of students' misconceptions are presented in the paper: modelling, prototype, overgeneralization, and process-object errors. Recognizing error type is crucial for planning the kind of mathematics instruction that will result in a higher level of comprehension. We will emphasize the role of productive tasks that trigger cognitive conflict and promote students' articulation, reformulation, reflection, and resolution of their thinking process. We present the results of recent studies on fractions focussing on typical misconceptions when dealing with different aspects of fractions. An analysis of errors gives us insight into the reasons for students' misconceptions, which may be related to a lack of conceptual understanding of fractions and the failure to recognize the mathematical context in a contextual, real-life situation. Research findings remind us of the importance of careful planning by teachers in order to promote the development of higher levels of comprehension in mathematics. Teacher should consider both the complexity of the mathematical concept and the importance of the associated mathematical context using an appropriate selection of realistic and contextual problems.

Keywords: misconceptions, fractions, productive task, modelling error, prototype error, overgeneralization, process-object error

Analiza napačnih predstav učencev kot metoda za izboljšanje poučevanja in učenja matematike z boljšim razumevanjem: primer ulomkov

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Povzetek. Napačne predstave učencev pri matematiki se lahko obravnavajo na različnih ravneh, ena izmed njih je prepoznavanje napak na globlji, diskurzivni ravni, ki zahteva diagnosticiranje razlogov za opažene napake in vključevanje učencev v dialog, razlago in utemeljevanje matematičnega razmišljanja. V prispevku so predstavljene štiri tipične vrste napačnih predstav učencev: napake pri modeliranju, prototipu, pretiranem posploševanju in procesno –objektna napaka. Prepoznavanje vrste napake je ključnega pomena za načrtovanje pouka matematike na način, ki bo usmerjen k višjim ravnem razumevanja. Poudarili bomo pomen produktivnih nalog, ki sprožajo kognitivni konflikt in spodbujajo učenca k artikuliranju, preoblikovanju, reflektiranju in razreševanju konfliktov lastnega miselnega procesa. Predstavljamo rezultate nedavnih študij o ulomkih, ki se osredotočajo na tipične napačne predstave pri obravnavanju različnih vidikov ulomkov. Analiza napak nam daje vpogled v razloge za napačne predstave učencev, ki so lahko povezane bodisi s pomanjkanjem konceptualnega razumevanja ulomkov bodisi z nezmožnostjo prepoznavanja matematičnega konteksta v realistični situaciji. Ugotovitve raziskav nas opozarjajo na pomen skrbnega učiteljevega načrtovanja pouka za spodbujanje razvoja višjih ravni razumevanja pri matematiki. Učitelj mora z ustrezno izbiro realističnih problemov naglasiti tako zapletenost matematičnega koncepta kot tudi pomen z njim povezanega matematičnega konteksta.

Ključne besede: napačne predstave, ulomki, produktivna naloga, napaka modeliranja, prototipska napaka, pretirano posploševanje, procesno-objektna napaka

Ways of discovering general rules of growing geometric patterns by students aged 15 to 17

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University of Split, Croatia

² Faculty of Educational Science, Juraj Dobrila University of Pula, Croatia

Abstract. The paper presents the results obtained in the first phase of collaborative action research on the development of algebraic and functional thinking in teaching and learning mathematics. In particular, different kinds of generalizations and ways of students' reasoning through the problem-solving process with growing patterns are considered.

In order to identify problems and get insight into students' knowledge and skills, an instrument was prepared, consisting of four tasks with growing geometric patterns and of different complexity (according to the type of functional rule and the manner of representation of the figures of which they are composed). The used tasks with growing patterns are stand-alone geometric problems because they are not explicitly related to any teaching unit and require a greater cognitive effort to determine the general rule and to connect different mathematical ideas and concepts.

A descriptive and qualitative analysis of the process of solving a stand-alone problem on a sample of two hundred students of Croatian urban high schools aged 15 to 17 was performed. The obtained results show that students mostly use figural reasoning, and only to a lesser extent numerical reasoning. Through the students' textual description, various forms of generalizations are observed, to a lesser extent carrying out symbolic generalization as well as setting a general rule.

Based on the obtained results, the second phase of action research is planned. Discussions and workshops on the role and use of (growing) geometric patterns will be conducted in collaboration with mathematics teachers, aimed at introducing changes in learning and teaching mathematics.

Keywords: action research, generalization, geometric pattern, stand-alone problem, visualization

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Načini otkrivanja pravila rastućih geometrijskih uzoraka učenika u dobi od 15 do 17 godina

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Sažetak. U ovom radu prikazuju se rezultati dobiveni u prvoj fazi kolaboracijskog akcijskog istraživanja o razvoju algebarskog i funkcijskog mišljenja u nastavi matematike. Posebno se razmatraju različiti načini zaključivanja i vrste generalizacija koje učenici izvode kroz proces rješavanja problema s rastućim uzorcima.

U svrhu identifikacije problema i uvida u znanja i vještine učenika, pripremljen je instrument s četiri zadatka s rastućim geometrijskim uzorcima, različite složenosti (prema vrsti funkcijskog pravila i prema načinu prikaza figura od kojih su sastavljeni). Korišteni zadaci s rastućim uzorcima predstavljaju izolirane geometrijske probleme jer nisu eksplicitno vezani niti uz jednu nastavnu cjelinu te je potrebno uložiti veći kognitivni napor za određivanje općeg pravila i povezati različite matematičke ideje i koncepte.

Deskriptivnom i kvalitativnom analizom obrađen je proces rješavanja izoliranog problema, na uzorku od dvjesto učenika hrvatskih gradskih srednjih škola u dobi od 15 do 17 godina. Dobiveni rezultati pokazuju da se učenici većim dijelom služe slikovnim zaključivanjem, a tek u manjoj mjeri broječanim. Kroz učenički tekstualni opis vidljivi su različiti oblici generalizacija pri čemu u manjoj mjeri provode simboličku generalizaciju kao i postavljanje općeg pravila.

Na temelju dobivenih rezultata planira se druga faza akcijskog istraživanja, u kojoj će se, u suradnji s nastavnicima matematike provesti rasprave i radionice o ulozi i načinu korištenja (rastućih) geometrijskih uzoraka s ciljem uvođenja promjena u proces učenja i poučavanja matematike.

Ključne riječi: akcijsko istraživanje, geometrijski uzorak, generalizacija, izolirani problem; vizualizacija

Spatial geometry in University entrance examinations in Hungary 1980-85 vs. 2015-2020

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Abstract. One of the outstanding challenges in technical higher education is to continue the development of visuospatial skills that has been started in primary and secondary education. In this lecture, the present state has been compared with the recent past in the light of the technical university entrance examinations, more precisely by comparing the spatial geometry problems occurring in the exams in both periods.

The question we set and try to answer is based on a subjective opinion having completed 25 years in technical higher education: is it true that the spatial abilities and geometric knowledge of incoming students are constantly narrowing?

Assessment is one of the basic concepts in the educational learning theory, which performs a feedback function in the examined pedagogical process. Its role is to measure the effectiveness of the teaching of a particular teaching unit and to observe whether the results meet the preliminary expectations. Adequate evaluation thus serves as a mirror for the whole pedagogical process. The utmost, macro-level, standardized, qualitative, summative, knowledge-level exam waiting for high school students is the matriculation, which has also been used as a university entrance examination in Hungary since 2005. Therefore, its use as an indicator of the secondary education process is valid.

Our research compares the written entrance examination tasks in Mathematics between 1980 and 85 with the advanced level written matriculation problems in Mathematics in the period 2015-20. The characteristics of teaching Mathematics and intermediate changes are also discussed.

In the study the quantity and weight of the spatial geometry problems in the two periods are compared and the visuospatial ability components required for their solution are identified, together with their quantity and complexity.

Keywords: visuospatial skills, higher education, spatial geometry, matriculation, teaching Mathematics

Térgeometria az egyetemi felvételi vizsgákon Magyarországon 1980-85 vs. 2015-20

Attila Bölcskei

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Absztrakt. A műszaki felsőoktatás egyik kitüntetett feladata az általános és középiskolai nevelésben megkezdett vizuális-téri képességfejlesztés továbbvitele. Jelen előadás célja az, hogy a műszaki egyetemi matematikai felvételi feladatok tükrében hasonlítsa össze a jelent a közelmúlttal, mégpedig a vizsgákban elforduló térgeometriai feladatok összehasonlításával.

A cél, annak a kérdésnek a vizsgálata, hogy a kutatás alátámasztja-e, azt a felsőoktatásban eltöltött több, mint 25 év oktatási tapasztalat nyomán megfogalmazódó szubjektív véleményt, hogy a bekerülő hallgatók téri képességei és geometriai ismeretei folyamatosan szűkülnek?

Az értékelés a tanítás-tanulási folyamat egyik alapfogalma, mely a vizsgált pedagógiai folyamatban visszacsatolási funkciót lát el. Szerepe az, hogy megvizsgáljuk, hogy egy adott tanítási egység tanítása mennyire sikeres, megfelelnek-e az előzetes várakozásoknak az eredmények? A jó értékelés tehát tükröként szolgál a teljes pedagógiai folyamat számára. A középiskolai diákokra váró legnagyobb, makroszintű, standardizált, kvalitatív, szummatív, tudásszint mérő vizsga az érettségi, mely Magyarországon 2005 óta egyúttal egyetemi felvételiként is szolgál. Fentiek indokolják, hogy a középiskolai oktatási folyamat indikátoraként használjuk.

A kutatás a fentiekben megfogalmazott kérdés vizsgálata céljából az 1980 és 85 közötti központi matematika írásbeli felvételiket hasonlítja össze a 2015-20 közötti emelt szintű írásbeli érettségi feladatokkal, kitérve az egyes időszakok matematika tanítási jellemzőire és az időközbeni változásokra.

A tanulmányban a térgeometriai feladatok mennyiségét és súlyát vetjük össze a két időszakban, illetve azonosítjuk a megoldásukhoz szükséges vizuális-téri képesség elemeket is és összehasonlítjuk ezek mennyiségét és komplexitását.

Kulcsszavai: vizuális-téri képességek, felsőoktatás, térgeometria, érettségi, a matematika tanítása

Potential of children math lesson for investigating peer interactions

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Abstract. Several studies in mathematics education look at the cooperation of students with each other. The focus is on partner or group work and its conditions (Götze 2007, Nührenböcker 2009, Schülke 2013, Gysin 2017). These are 'peer situations'. A very special form of student participation is offered by children's math lessons, which are characterized by one or two students 'planning', structuring, and leading the lesson, while the teacher largely observes the lesson. The available data on such a children's math lesson analyzed interpretatively-qualitatively in order to work out particularities in the interaction and the distribution of roles of the students. The leading students take on "double roles" in the present data. On the one hand, they act as teachers and are perceived as such by their classmates. On the other hand, they act as equal peers. Increasingly, questions can be reconstructed that critically and constructively question the technical procedure of written multiplication. Such children's questions are considered to be interesting and productive by researchers (Götz 1997), but according to Ritz-Fröhlich (1992) they occur very rarely in the classroom, addressed to the teacher. The extent to which such children's questions are encouraged and supported by the format of the children's math lesson is investigated based on the present data.

Keywords: children's math lesson, children's questions, peer situation, roles, language

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Kindermathematikstunde – Untersuchung der Potentiale von Peer-Interaktionen

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Abstrakt. In der Mathematikdidaktik gibt es mehrere Studien, die Zusammenarbeit von Schüler*innen untereinander betrachten. Dabei stehen Partner- oder Gruppenarbeiten und deren Bedingungen im Mittelpunkt (Götze 2007, Nührenböcker 2009, Schülke 2013, Gysin 2017). Es handelt sich um ‚Peer-Situationen‘. Eine ganz besondere Form der Mitgestaltung von Schüler*innen bieten Kindermathematikstunden, die dadurch gekennzeichnet sind, dass ein oder zwei Schüler*innen den Unterricht ‚planen‘, strukturieren und die Leitung der Stunde übernehmen, während die Lehrperson größtenteils beobachtend am Unterrichtsgeschehen teilnimmt. Die vorliegenden Daten zu einer solchen Kindermathematikstunde werden interpretativ-qualitativ analysiert, um Besonderheiten in der Interaktion und der Rollenverteilung der Schüler*innen herauszuarbeiten. Die leitenden Schülerinnen übernehmen in den vorliegenden Daten „Doppelrollen“. Auf der einen Seite agieren sie als Lehrende und werden von ihren Mitschüler*innen auch so wahrgenommen. Auf der anderen Seite agieren sie als gleichberechtigte Peers. Es lassen sich vermehrt Fragen rekonstruieren, die kritisch-konstruktiv das fachliche Verfahren der schriftlichen Multiplikation hinterfragen. Solche Kinderfragen werden von Forschenden als fachlich interessant und produktiv angesehen (Götz 1997), treten jedoch laut Ritz-Fröhlich (1992) im Klassenunterricht, gerichtet an die Lehrperson, sehr selten auf. Inwieweit solche Kinderfragen durch das Format der Kindermathematikstunde begünstigt und unterstützt werden, wird anhand der vorliegenden Daten untersucht.

Schlüsselwörter: Kinder-Mathematikstunde, Kinderfragen, Peer-Situation, Rollen, Sprache

The role of mathematics in the successful acquisition of teaching material in physics

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Abstract. Physics and mathematics have been deeply interwoven throughout the long history of science; nevertheless they represent two separate sciences. Physics teachers often claim that their students do not understand physics as a result of mathematical knowledge deficiency, and that mathematical knowledge guarantees successful learning of physics (Pietrocola, 2008). Although mathematics is the “language” of physics, it still fails to coincide with the one used in mathematics teaching (Redish, 2006). Could the way of math teaching contribute to the understanding of physics concepts, and, if so, in which way? The trend towards more formal and abstract teaching is noticeable in secondary school math and physics teaching. Specifically, physics teaching fails to address the explanation of basic concepts, whereas math teaching is increasingly abstract, without stressing its application. Prerequisite of successful math teaching is based on continuous relating math and other sciences. If harmony between math and other sciences has not been reached, there will be no success in achieving set objectives neither in math teaching nor in other areas where its application is necessary. Physics is an excellent place for students to learn how to use math, but too strong emphasis on algorithmic approaches could block students from installing knowledge into the intuition and understanding of physics terms. Our research has shown that math knowledge is taken down to calculating without practical application, thus the acquired knowledge fails to contribute to easier and more simplified adoption of teaching material in physics teaching. Successful physics teaching is grounded on the adequate application of math apparatus as well as on the correct understanding of basic physics concepts.

Keywords: mathematics, physics, mathematical algorithm, physics concept

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Uloga matematike u uspješnom usvajanju nastavnih sadržaja iz fizike

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Sažetak. Fizika i matematika su duboko isprepleteni kroz dugu povijest znanosti, ali ipak predstavljaju dvije odvojene znanosti. Učitelji fizike često tvrde da njihovi učenici ne razumiju fiziku kao rezultat nedostatka matematičkog znanja, te da matematičko znanje garantira uspješno učenje fizike (Pietrocola, 2008). Iako matematika jeste "jezik" fizike ipak se ne poklapa s onim koji se koristi u nastavi matematike (Redish, 2006). Može li način podučavanja matematike doprinijeti razumijevanju fizičkih pojmova i ako može, na koji način? Primjetan je trend u srednjoškolskoj nastavi matematike i fizike prema formalnijem i apstraktnijem izlaganju. Poučavanje fizike se posebno ne osvrće na objašnjenje temeljnih koncepata, dok je poučavanje matematike sve apstraktnije bez naglaska na primjeni. Preduvjet uspješne nastave matematike upravo se temelji na stalnom povezivanju matematike i drugih znanosti. Ukoliko nije postignut sklad između matematike i drugih znanosti s uspjehom se neće postići zadani ciljevi ni u nastavi matematike, kao ni u drugim oblastima gdje je njena primjena neophodna. Fizika je izvrsno mjesto za učenike da nauče koristiti matematiku, ali preveliki naglasak na algoritamskim pristupima može blokirati učenike da ugrađuju znanje u intuicije i razumijevanje fizičkih pojmova. Naša istraživanja su pokazala da se znanje matematike svodi na računanje bez praktične primjene, pa stečeno znanje ne doprinosi lakšem i jednostavnijem usvajanju gradiva u nastavi fizike. Uspješna nastava fizike se temelji na adekvatnoj primjeni matematičkog aparata kao i na ispravnom razumijevanju temeljnih fizikalnih koncepata.

Ključne riječi: matematika, fizika, korelacija, matematički algoritam, fizički koncept

Interdisciplinarity in mathematics teacher education

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Abstract. Interdisciplinary approach plays an important role in the development of a student profile based on 21st century skills. The implementation of interdisciplinarity is a challenge for both teachers and teacher educators. It is essential for a teacher to be aware of the space of interdisciplinary ideas to shift the vector of school education from subject-oriented to interdisciplinary one. How ready are students (prospective mathematics teachers) to use an interdisciplinary approach in teaching schoolchildren? The aim of this study is to create an interdisciplinary model for mathematics teacher education; in particular, we are interested in an interdisciplinary approach to teaching mathematics and computer science. We conducted an empirical study with students 2-4 years of study. In the classes on the subject "Fundamentals of Research in the Field of Mathematical Education", students studied and solved interdisciplinary tasks from the field of mathematics and computer science at the school level. Students (future teachers) were able to see the connection of mathematics with other subjects; they expressed their interest and desire to develop lessons for schoolchildren on the topics "mathematics and artificial intelligence", "mathematics and data analysis", etc.

Keywords: interdisciplinary approach, teacher education, research tasks, mathematics, computer science

МЕЖДИСЦИПЛИНАРНОСТЬ В ОБУЧЕНИИ УЧИТЕЛЯ МАТЕМАТИКИ

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Абстрактный. Междисциплинарный подход играет важную роль в образовании студента, основанного на навыках 21 века. Реализация междисциплинарности является проблемой как для учителей, так и для преподавателей университетов. Учителю необходимо знать междисциплинарные идеи, чтобы сместить вектор школьного образования с предметного на междисциплинарный. Насколько готовы студенты (будущие учителя математики) использовать междисциплинарный подход в обучении школьников? Целью данного исследования является разработка междисциплинарной модели обучения учителей математики; в частности, нас интересует междисциплинарный подход к обучению математике и информатике. Мы провели эксперимент со студентами 2-4 курсов. На занятиях по дисциплине «Основы исследований в области математического образования» студенты изучали и решали междисциплинарные задачи из области математики и информатики на школьном уровне. Студенты (будущие учителя) смогли увидеть связь математики с другими предметами; они выразили заинтересованность и желание разрабатывать уроки для школьников по темам «математика и искусственный интеллект», «математика и анализ данных» и др.

Ключевые слова: междисциплинарный подход, педагогическое образование, исследовательские задачи, математика, компьютерные науки

Fluency of mental computation of a prospective teachers

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Abstract. According to (Kilpatrick, 2001), procedural fluency, as one of the five elements of mathematical proficiency, refers to the flexible, precise, efficient ability to perform procedures appropriately. In the part of arithmetic, procedural fluency refers to the automation of computation, mental computation and written computation. (Star, 2000) argues that an in-depth understanding of mathematics is not possible without the acquisition of basic facts and procedural computational skills. Teacher education students are not basically oriented towards mathematics, and during schooling, due to the frequent use of calculators, they do not develop thinking skills that are, among other things, of great importance for the quality performance of primary school teachers. This paper presents a survey on the skill of mental arithmetic in students of teacher education from the first to the fifth year. Sample of the survey consist of 139 students of teacher education and the results showed that there is a statistically significant difference between students of lower and higher years of study. The difference in success is attributed to active work with senior students on achieving fluency and raising awareness of the importance of mental calculation, as well as strengthening the use of different strategies.

Keywords: procedural knowledge, conceptual knowledge, mental computation, strategies, fluency

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Fluentnost misaonog računanja budućih učitelja

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Sažetak. Prema Kilpatrick (2001) proceduralna fluentnost, kao jedan od pet elemenata matematičkog umijeća, odnosi se na fleksibilnu, preciznu, učinkovitu sposobnost obavljanja postupaka na odgovarajući način. U dijelu aritmetike proceduralna fluentnost odnosi se na automatizaciju računanja, misaono računanje i pisano računanje. Star (2000) smatra da dubinsko razumijevanja matematike nije moguće bez usvajanja osnovnih činjeničnih znanja i proceduralnih računskih vještina. Studenti učiteljskog studija nisu bazično orijentirani prema matematici, a tijekom školovanja, zbog česte upotrebe računala, ne razvijaju vještine misaonog računa koje su im, između ostalog, od velike važnosti za kvalitetno obavljanje posla učitelja razredne nastave. U ovom radu bit će prezentirano istraživanje o vještini misaonog računa kod studenata učiteljskog studija od prve do pete godine. U istraživanju je sudjelovalo 139 studenata učiteljskog studija, a rezultati su pokazali da postoji statistički značajna razlika između studenata nižih i viših godina studija. Razlika u uspjehu pripisuje se aktivnom radu sa studentima viših godina na postizanju fluentnosti i osvještavanju važnosti misaonog računa, kao i jačanju upotrebe različitih strategija.

Ključne riječi: proceduralno znanje, konceptualno znanje, misaono računanje, strategije, fluentnost

Does online learning make a difference in students' grades?

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Abstract. This past academic year was challenging for both students and professors. Although the Covid 19 pandemic led to a chaotic 2020, it will also greatly affect the years to come. While many universities are now making plans for future distance learning, it is a good time to reevaluate the pros and cons of online learning. The purpose of this paper is to determine if there is a difference in student grades between online and face-to-face learning by comparing exam scores in Mathematics 1 and Mathematics 2 in the academic year in which both courses were taken at the faculty with the year in which Mathematics 1 was taken at the faculty and Mathematics 2 was taken in a distance learning environment. The statistical indicators obtained suggest that the pass rate is almost the same regardless of the form of teaching and midterm exams. However, the correlation coefficient between the results of the exams of Mathematics 1 and Mathematics 2 courses is statistically significantly lower in the academic year in which the first course was taken at the faculty and the second in a distance learning environment. This suggests that there are a certain number of students who did not perform well in the online environment, but also those who were helped to perform better. Through the method of interviewing students, both models have uncovered key factors in the teaching process that are believed to be the main causes of student success or failure.

Keywords: online learning, face-to-face learning, grades, correlation, mathematics

Postoji li razlika u ocjenama studenata kada se nastava održava na daljinu?

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Sažetak. Prethodna akademska godina bila je izazovna, kako za studente tako i za profesore. Pandemija uzrokovana virusom Covid 19 uzrokovala je kaotičnu 2020. godinu te će utjecati i na naredne godine. Mnoga sveučilišta rade planove za nastavak učenja na daljinu te je sada pravo vrijeme za evaluaciju nastave u online okruženju, njenih prednosti i nedostataka. Cilj ovog rada je ispitivanje postojanja razlika u uspjehu studenata kada se nastava održava u učionicama od učenja u online okruženju. Uspoređuju se ocjene kolegija Matematika I i Matematika II u akademskoj godini kada se nastava iz oba predmeta izvodila u učionicama i u godini u kojoj se nastava iz kolegija Matematika I održavala u učionici, a iz kolegija Matematika II na daljinu. Dobiveni statistički pokazatelji ukazuju da je prolaznost gotovo ista bez obzira na oblik izvođenja nastave. Međutim, koeficijent korelacije za ocjene iz kolegija Matematika I i Matematika II je statistički značajno manji kada se nastava iz prvog predmeta održavala u učionicama, a iz drugog u online okruženju, što ukazuje na postojanje određenog broja studenata koji se nisu dobro snašli u online okruženju, kao i onih kojima je isto pomoglo da ostvare bolje rezultate. Metodom intervjua sa studentima došlo se do ključnih čimbenika nastavnog procesa u oba modela za koje se pretpostavlja da su glavni uzroci studentskog (ne)uspjeha.

Ključne riječi: učenje na daljinu, nastava u učionici, ocjene, korelacija, matematika

Visualization in the teaching and learning of mathematics

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Abstract. Teaching mathematics requires the use of various external representations: concrete models (objects) and visual models (schematic drawings, pictures). The activity is much more efficient if pupils employ and combine several representations at the same time.

This presentation focuses on a pedagogical experiment assessing the problem solving skills of elementary school children and teacher training students. The experimental and control groups solved the problems using various visual representations. We hypothesized that using different representations facilitates problem solving to various extent. The hypothesis was proved as those participants who used cards were more likely to devise strategies and achieved better compared to the participants who used virtual cards.

Keywords: mathematical competence, problem-solving skills, visualization, representations, mathematical model

Vizualitás a matematika oktatásában és tanulásában

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Absztrakt. Többféle külső reprezentáció, konkrét (tárgyi) és vizuális (képi) modell alkalmazása szükséges a matematika tanításakor, a tevékenység sokkal hatékonyabb, ha a tanuló többféle reprezentációt párhuzamosan használ és összekapcsolja azokat.

Az előadásban egy pedagógiai kísérletről számolok be, amely során elemi osztályos tanulók és tanítóképzésben résztvevő hallgatók problémamegoldó képességét mértük, a feladatokat különböző vizuális reprezentációkat használva oldotta meg a kísérleti csoport, illetve a kontroll csoport. A hipotézisünk az volt, hogy a különböző reprezentációk alkalmazása eltérő mértékben lehet segítségükre a megoldásban. A kísérlet során ez be is bizonyosodott, hisz azok, akik kártyákkal dolgozhattak, nagyobb eséllyel dolgoztak ki stratégiát, jobban teljesítettek, mint azok, akik virtuális kártyákkal dolgozhattak.

Kulcsszavak: matematikai kompetencia, problémamegoldás, vizualitás, reprezentációk, matematikai modell

eTwinning as a potential tool in teacher education

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Abstract. eTwinning is a segment of the Erasmus program which seeks to utilize new technologies in order to network schools and kindergartens. Next to its emphasis on the importance of digital competencies, eTwinning also underlines the importance of collaborative and project-based work, as well as lifelong learning in a safe environment for all participants in the online environment. eTwinning launched a pilot program Teacher Training Institutions (TTIs) in 2012 aimed to include teacher education institutions in its platform. Since 2018, the TTIs initiative has been officially available to all countries participating in the eTwinning program. Faculty of Education in Osijek has been participating in the TTIs initiative since 2014 and has carried out nine projects so far. Given that the Croatian National Curriculum Framework emphasizes the importance of moving away from traditional educational models and places emphasis on the development of critical thinking skills, creativity, innovation, communication skills, collaboration, informational and digital literacy, eTwinning offers the possibilities of new modes of work aimed to develop these competencies within its platform. This paper will present the results of a survey conducted among students of the Faculty of Education in Osijek related to their familiarity with eTwinning, opinions on advantages and disadvantages of implementing eTwinning in regular education, as well as their attitudes on the possibility of applying eTwinning in school subjects Mathematics and Computer Science.

Keywords: eTwinning, competencies, Mathematics, ICT, teacher education

eTwinning kao opcija u obrazovanju budućih učitelja

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Sažetak. eTwinning je sastavni dio Erasmus programa koji teži umrežavanju škola i vrtića uz pomoć novih tehnologija. Osim isticanja važnosti digitalne kompetencije, eTwinning ističe i važnost suradničkog i projektnog rada te cjeloživotnog učenja u sigurnom okruženju svih sudionika na internetu. Od 2012. godine eTwinning je započeo s pilot programom uključivanja ustanova za obrazovanje budućih učitelja u svoju zajednicu pod nazivom Teacher Training Institutions (TTIs), a od 2018. godine je TTIs inicijativa službeno dostupna svim zemljama koje sudjeluju u eTwinningu. Fakultet za odgojne i obrazovne znanosti u Osijeku sudjeluje u TTIs inicijativi od 2014. godine te je do sada ostvareno devet projekata. Kako je u Okviru nacionalnoga kurikulumata istaknuta važnost u odmičanju od tradicionalnoga načina obrazovanja te se stavlja naglasak na razvoj kritičkog mišljenja, kreativnosti i inovativnosti, komunikacijskih vještina, suradnje, informacijske i digitalne pismenosti, eTwinning kroz svoju platformu nudi mogućnost novih oblika rada u svrhu razvoja istaknutih kompetencija. U radu će biti dani rezultati provedenog upitnika među studentima Fakulteta za odgojne i obrazovne znanosti u Osijeku o upoznatosti s eTwinningom, stavovima o prednostima i nedostacima implementacije eTwinning-a u redovnu nastavu kao i stavovima o mogućnostima primjene u kontekstu matematičkih i informatičkih predmeta.

Ključne riječi: eTwinning, kompetencije, matematika, IKT, obrazovanje učitelja

Transformation of population density data in the Republic of Croatia

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Abstract. The paper presents the transformation of population density data of the Republic of Croatia from the *Mid-year total population estimate of the Republic of Croatia, by counties, 2019*, and a digitized map of the Republic of Croatia with 21 complexly connected territorial units (20 counties and the City of Zagreb) into a simple model of 16 zones of uniform population density and different sizes, proportional to the number of inhabitants therein.

Data transformation is carried out in five steps; determining the optimal circular section on a digitized map, dividing it into an arbitrary number of matching circular sections (16 in the paper), and determining the intersection of each with county faces by counting common pixels. The inhabitants of the counties are added to the zones in proportion to their share in the area of the county. Finally, a new system of zones of uniform density, different sizes, and proportional to the population therein, is created.

Keywords: digitized map, pixel, data transformation, intersection, ratios

Transformacija podataka o naseljenosti Republike Hrvatske

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Sažetak. Rad donosi prikaz transformacije podataka o gustoći naseljenosti Republike Hrvatske iz *Procjene ukupnog stanovništva Republike Hrvatske po županijama sredinom godine 2019.* i digitalizirane karte Republike Hrvatske s 21 kompleksno povezanom teritorijalnom cjelinom (20 županija i grad Zagreb) u jednostavan model od 16 zona jednolike gustoće naseljenosti, različitih veličina, razmjernih broju stanovnika u njima.

Transformacija podataka odvija se u pet koraka. Određivanje optimalnog kružnog isječka na digitaliziranoj karti, njegova podjela na proizvoljan broj sukladnih kružnih isječaka (16 u radu), te određivanje presjeka svakoga s ploham županija prebrojavanjem zajedničkih piksela. Razmjerno udjelu u površini županije zonama se pribrajaju stanovnici županija. Završno se stvara novi sustav zona jednolike gustoće, različitih veličina, razmjernan populaciji u njima.

Ključne riječi: digitalizirana mapa, piksel, transformacija podataka, presjek skupova, omjeri

Mathematics teachers as popularizers of science

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Abstract. As teachers are a significant part of student's education, can they define the student's main interests? Do mathematics teachers need to popularize just mathematics or science in general? Will that help students become mathematicians or researchers? How can mathematics teachers be a strong connection between students and scientists? Is this even important for all the new research we wish to have in the future? How can teachers inspire students? How can they help them to increase their curiosity? There are a lot of questions we can ask, or we can just act. In this presentation, I will present examples of how mathematics teachers can help students engage in science through school projects or EU projects. I will also present what kind of education (training) can help teachers to keep them inspired and even enhance their motivation throughout their teaching career. It would be useful to know if teachers' inspiration as popularizers of science can motivate students to become scientists. Furthermore, as researchers, we can prepare more resources to support in-service teachers in their popularization of science through open science days. We can also educate them how to use resources. We can motivate them to make their own resources, or they can motivate and help their students to make their own resources or project assignments. I will show you an example about the well-known Monty Hall Problem which students have made together with a mathematics teacher.

Keywords: popularization of science, scientists, teacher education, students' assignments, probability

Učitelji matematike kao popularizatori znanosti

Marina Furkes

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Sažetak. Kako su učitelji važan dio učenikova obrazovanja, mogu li oni predodrediti učenikove glavne interese? Moraju li učitelji i nastavnici matematike popularizirati samo matematiku ili znanost općenito? Hoće li navedeno pomoći učenicima da postanu matematičari ili istraživači? Kako učitelji i nastavnici matematike mogu postati poveznica između učenika i znanstvenika? Smatramo li to kao nešto važno i potrebno za buduća istraživanja? Kako učitelji mogu inspirirati učenike? Kako im mogu pomoći u podizanju njihove znatiželje? Možemo postavljati mnoga pitanja ili možemo djelovati. U ovoj prezentaciji navest ću primjere kako učitelji i nastavnici matematike mogu pomoći učenicima da konzumiraju znanost kroz školske i EU projekte. Također ću prezentirati koja vrsta edukacije je potrebna učiteljima u zadržavanju i obnavljanju njihove inspiracije tijekom učiteljske karijere. Bilo bi korisno znati utječe li inspiracija učitelja kao popularizatora znanosti na motivaciju učenika da i sami postanu znanstvenici. Štoviše, kao istraživači možemo pripremiti materijale kako bismo podržali učitelje i pomogli im u kreiranju otvorenih dana znanosti tijekom kojih populariziraju znanost učenicima i širem građanstvu. Možemo ih educirati kako koristiti materijale ili ih motivirati u izradi vlastitih. Isto tako učitelji mogu motivirati i pomoći svojim učenicima u izradi učeničkih projektnih zadataka. Na kraju prezentacije pokazat ću jedan matematički primjer projektnog zadatka pod nazivom „Monty Hall Problem“ koji su osmislili i izradili učenici zajedno sa svojom nastavnicom matematike.

Ključne riječi: popularizacija znanosti, znanstvenici, edukacija nastavnika, učenički projektni zadaci, vjerojatnost

Number representation in textbooks for primary mathematics education in Croatia

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Abstract. One of the key competencies for everyday life is working with numbers. Since the number is an abstract notion that signifies the common property of equipotent sets, in mathematics education, the notion is developed from concrete experience, through a pictorial view toward an abstract idea. Number representation has an essential role in the process. Understanding, using and interpreting different representations is a relevant component of number sense. The Croatian mathematics curriculum includes representing numbers in different ways. Considering that primary school teachers in Croatia use textbooks in teaching mathematics, it is relevant to recognise the number representation in textbooks. We explored which representations of numbers are used in the textbooks for the first three primary school grades from different publishers aligned with the curriculum. The results revealed various representations across textbooks. The cardinality of a particular set is a dominant number representation. Textbooks rarely suggest a concrete experience, use a few representations, and skip ahead to the abstract idea of the number. It is a teacher's responsibility to organise concrete experience and implement representation from textbooks of different editions.

Keywords: mathematics textbook, number representation, number sense, primary mathematics education

Prikazi broja u udžbenicima za razrednu nastavu matematike u Hrvatskoj

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Sažetak. Razvijanje kompetencija u radu s brojevima je ključno za svakodnevni život. Kako je broj apstraktan pojam kojim se označava zajedničko svojstvo ekvipotentnih skupova, u nastavi matematike pojam broja razvijamo polazeći od konkretnog iskustva, preko slikovnog prikaza do apstraktne ideje. Važnu ulogu pri tom imaju prikazi (reprezentacije) broja. Razumijevanje, korištenje i tumačenje različitih reprezentacija broja bitna je komponenta osjećaja za broj. Hrvatski kurikulum za matematiku u razradi ishoda uključuje prikazivanje brojeva na različite načine. Uzevši u obzir da učitelji razredne nastave u Hrvatskoj koriste udžbenike u nastavi matematike bitno je poznavati reprezentacije brojeva u udžbenicima. Istražili smo kakve reprezentacije broja se koriste u udžbenicima za prva tri razreda osnovne škole različitih izdavača čije je izdanje usklađeno s ishodima kurikuluma. Rezultati su pokazali da se u udžbenicima daju različite reprezentacije broja. Dominantan je prikaz broja kao kardinalnog broja nekog skupa elemenata. Udžbenici rijetko sugeriraju konkretno iskustvo, koriste manji broj reprezentacija i rano prelaze na apstraktno poimanje broja. Odgovornost je učitelja organizirati konkretno iskustvo i implementirati reprezentacije prisutne u udžbenicima različitih izdanja.

Ključne riječi: matematički udžbenik, osjećaj za broj, prikazi broja, razredna nastava matematike

Designated geometry curriculum in Croatian primary education: A critical view

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Abstract. Even though geometry has always been an important part of mathematics, this was not necessarily the case with geometry having an important place in mathematics education. The decrease of geometry content within the mathematics curricula all over the world in the past decades has highlighted the discussions on the importance of learning geometry, particularly because of its both visual and abstract aspects. These discussions encompassed the questions on relevant geometry content, its coherence, and learning trajectories across the curriculum. A possible first step in approaching this issue is getting an insight into its designated curriculum. The study presented here refers to investigating the fundamental ideas of geometry within the Croatian curricula from 2006 and 2019 for the primary levels. The content requirements from the standpoint of fundamental ideas and accompanying learning trajectories were investigated for each curriculum separately, but also regarding the coherence between the old and new curricula. The results show that both 2006 and 2019 curricula focus primarily on two fundamental ideas of geometry (i.e., geometric forms and their construction, measurement), whilst the other five are of small importance, if at all. Both curricula disregard certain core contents important in primary geometry education, such as symmetry and spatial visualization. Further results will be presented in the presentation. This study points to the importance of reflecting on the geometry issues as a preparation for a coherent and well-designed national geometry curriculum for primary education.

Keywords: designated curriculum, geometry education, fundamental ideas, learning trajectories, content analysis

Kritički pogled na hrvatski planirani kurikulum za geometriju u razrednoj nastavi

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Sažetak. Iako je geometrija oduvijek bila važan dio matematike, nastava geometrije nema ili nije uvijek imala taj značaj i važnost unutar nastave matematike. Smanjenje geometrijskih sadržaja u matematičkim kurikulumima diljem svijeta u prošlim desetljećima je potaknulo diskusije o važnosti učenja geometrije, posebice vezano uz njene kako vizualne tako i apstraktne aspekte. Ove diskusije su obuhvatile i pitanja o relevantnim geometrijskim sadržajima, njihovoj koherentnosti te trajektorijama učenja kroz kurikulum. Mogući prvi korak u prilaženju ovim pitanjima je uvid u planirani kurikulum. Studija prikazana u ovom radu se odnosi na ispitivanje fundamentalnih ideja geometrije unutar hrvatskih dokumenata Nastavnog plana i programa iz 2006. godine i Kurikuluma za nastavni predmet matematike iz 2019. godine za razinu razredne nastave. Sadržajni zahtjevi su ispitani s gledišta fundamentalnih ideja i pripadajućih trajektorija učenja, i to zasebno za svaki od spomenutih dokumenata, ali i gledajući veze između starog i novog dokumenta. Rezultati pokazuju da se oba dokumenta prvenstveno fokusiraju na dvije fundamentalne ideje geometrije (tj. geometrijske oblike i njihovu konstrukciju te mjerenje), dok su ostalih pet ideja od malog značaja ili bez njega. Oba dokumenta zanemaruju određene ključne sadržaje važne za razrednu nastavu geometrije, poput simetrije i prostornog zora. Daljnji rezultati bit će predstavljeni u prezentaciji. Ova studija ukazuje na važnost promišljanja o geometriji kao pripremi za koherentan i dobro dizajniran kurikulum geometrije za razrednu nastavu.

Ključne riječi: planirani kurikulum, nastava geometrije, fundamentalne ideje, trajektorije učenja, sadržajna analiza

Essay as technique of assessing the element of mathematical communication

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Abstract. Mathematical communication, as an element of knowledge assessment in mathematics, is recently introduced by curricular reform in Croatia. The objective of assessing mathematical communication is to evaluate how students use mathematical language, ask questions and elaborate answers, use mathematical symbols and read algorithms, alternate between data representations and apply technology. One of many arising questions, due to this matter, are techniques most suitable for the investigation of students' achievements in using apt mathematical language. In this paper, researchers are analysing mathematical essay in order to investigate whether it would suit the requirements for assessing mathematical communication. Researchers selected mathematical essay since it demands correlation of knowledge, description, elaboration, everyday life examples, drawing, modelling and alternative data representations.

Keywords: mathematical communication, mathematical essay, mathematical language, assessment, summative assessment

Esej kao način vrednovanja matematičke komunikacije

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Sažetak. Kurikularnom reformom obrazovanja u Hrvatskoj, popularno nazvanom „Škola za život“ uvedeni su novi elementi vrednovanja. Jedan od tih elemenata, u matematičkom području, je matematička komunikacija. Uvođenjem matematičke komunikacije želi se evaluirati; učeničko korištenje matematičkog jezika, sposobnost postavljanja pitanja te obrazlaganja odgovora, korištenje i uporaba matematičkih simbola te čitanje algoritama, vještinu prelaženja s jedne vrste prikazivanja podataka na drugu i na kraju korištenje tehnologije. No, na nesigurnost nailazimo pri odabiru primjerenih tehnika koje će na najučinkovitiji način provjeriti usvojenost ishoda matematičke komunikacije. U ovom istraživanju, analizira se i proučava matematički esej kako bi utvrdilo zadovoljava li on svim uvjetima za vrednovanje matematičke komunikacije. Matematički esej izabran je za detaljnu analizu jer zahtjeva povezivanje znanja, opisivanje, obrazlaganje, povezivanje s primjerima iz svakodnevnog života, skiciranje te različite prikaze podataka.

Cljučne riječi: matematička komunikacija, matematički esej, matematički jezik, vrednovanje, sumativno vrednovanje

Beyond dichotomies in mathematics teaching

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Abstract. In school instruction, including mathematics teaching, we have long observed that heterogeneous social and educational processes result in “uncertainty” among teachers with regard to their professional knowledge and to processes in which theories and concepts, rather than being applied reflectively, are transformed into popular “ideas” and “solutions” that lead away from quality pedagogical work. Our starting point is the thesis that the quality professional knowledge of teachers and its application in mathematics teaching requires the permanent assertion of the teacher’s autonomy of thinking and his/her opposition to various “external” ideologies and “pedagogical errors”.

It is not, of course, possible to exhaust all of the pedagogical errors that constantly arise in the school field in the present theoretical paper. Instead, we will focus on the following dichotomies: abstract/concrete, repetition in learning/learning as fun, and the transfer of knowledge/the independent discovery of knowledge. A common feature of all such dichotomies is that they each contain one part that is a priori negatively marked as something “traditional”, “surviving”, “bad” or “overly demanding”.

We demonstrate why teaching must take place beyond dichotomies and show the complexity of the pedagogical process with reference to the goals established in lesson plans, professional and pedagogical knowledge, and individual examples in the field of teaching and learning mathematics. The operation of the aforementioned dichotomies is primarily placed in the context of simplistically perceived constructivism. Constructivism, as defined, for example, by one of its founders, Glaserfeld, is not prescriptive but descriptive. It does not instruct the teacher on how to act in the classroom, but rather encourages him/her to think about how to lead the learning process, which, given its complexity, can clearly not be fully controlled by

the teacher. For instance, the idea that “students create their own knowledge”, which is a widespread understanding of constructivism, omits basic assumptions about knowledge and learning; namely, that knowledge is a social construct (the goals and standards of knowledge cannot, therefore, be defined by the student and, taken as a whole, instruction is the transfer of knowledge) and that learning is the formation (construction) of internal representations (structures) on the basis of external representations and the constant intertwining and supplementing of both.

Since it is realistic to assume that the “creativity” of pedagogical “innovators” will not simply disappear one day, the pursuit of quality in teaching and in students’ knowledge demands professional didactic knowledge from teachers and the courage to “step aside”, that is, an ethics that rejects misguided pedagogical dichotomies that result in teachers who, under the slogans of the “creativity”, “independent discovery” and “innovation” of the student, deviate from structured mathematical knowledge.

Keywords: teaching mathematics, “pedagogical errors”, dichotomies, constructivism

Onstran dihotomij pri poučevanju matematike

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Povzetek. Pri pouku v šoli in tudi pri poučevanju matematike že dalj časa opazamo, da heterogeni družbeni in edukacijski procesi vodijo v učiteljevo »negotovost« strokovnega znanja in v procese, v katerih se teorije in koncepte namesto reflektirane uporabe pretvarja v popularne »pojme« in »rešitve«, ki pedagoško delo učiteljev vodijo stran od kakovosti. Izhajamo iz teze, da kakovostno strokovno znanje učiteljev in njegova uporaba pri poučevanju matematike zahteva permanentno uveljavljanje učiteljeve avtonomije mišljenja in njegovo zoperstavljanje različnim »zunanjam« ideologijam in »pedagoškim zmotam«.

V teoretičnem prispevku seveda ne moremo izčrpati vseh pedagoških zmot, ki nenehno vznikajo v šolskem polju, posvetili se bomo naslednjim dihotomijam: abstraktno/konkretno, ponavljanje v učenju/učenje kot zabava in prenos znanja/samostojno odkrivanje znanja. Skupna poteza vseh takšnih dihotomij je, da je v vsaki posebej en del a priori vrednotno označen negativno: označen je kot nekaj »tradicionalnega«, »preživelega«, »slabega«, »prezahtevnega«.

V prispevku pokažemo, zakaj mora poučevanje potekati onstran dihotomij, in, izhajajoč iz v učnih načrtih postavljenih ciljev, strokovnega in pedagoškega znanja, na posameznih primerih na področju poučevanja in učenja matematike pokažemo kompleksnost pedagoškega procesa. Delovanje prej navedenih dihotomij postavljamo predvsem v kontekst poenostavljeno dojetega konstruktivizma. Konstruktivizem, kot ga denimo opredeli eden od utemeljitev Glasersfeld, ni predpisujoč, ampak opisujoč. Ne podaja navodil učitelju, kako naj ravna v razredu, spodbuja ga k razmisleku, kako voditi proces učenja, za katerega je glede na njegovo kompleksnost jasno, da ga ne more popolnoma nadzorovati. Denimo, razširjeno razumevanje konstruktivizma »učenci kreirajo lastno znanje« izpušča temeljne predpostavke o znanju in o učenju, namreč da je znanje družbeni konstrukt (zato ciljev in standardov znanja ne more opredeliti sam učenec in pouk kot celota je

prenos znanja) in da je učenje oblikovanje (konstruiranje) notranjih reprezentacij (struktur) na osnovi zunanjih reprezentacij in nenehnega prepletanja ter dopolnjevanja obojih.

Ker je realistično predpostaviti, da »kreativnost« pedagoških »inovatorjev« ne bo preprosto nekega dne izginila, prizadevanja za kakovost v poučevanju in v znanju učencev od učiteljev zahtevajo strokovno didaktično znanje in pogum »koraka v stran«, to je etiko, ki zavrača zgrešene pedagoške dihotomije, katerih posledica je poučevanje učiteljev, ki pod gesli »ustvarjalnosti«, »samostojnega odkrivanja«, »inovativnosti« učenca odmika od strukturiranega matematičnega znanja.

Ključne besede: poučevanje matematike, »pedagoške zmote«, dihotomije, konstruktivizem

Exploring the factors that influence the utilization of digital games in the mathematics classroom

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Abstract. Nowadays, teachers experience great difficulties in motivating students and engaging them in the teaching-learning process. A good way to increase students' participation in the classroom is by using purposeful games. Mathematical games have become an interesting area in educational research because they develop the interest in learning mathematics through anticipation of competition, challenge, and fun. Games can function as individual learning activities or as a powerful content delivery mechanism over several sessions. For a game to be effective, it must align with learning outcomes and should not be competitive in all its aspects. Sometimes the game might require students to work collaboratively to solve problems, while in other contexts, the game might make students compete against one another to reach a personal best. The study presented in this paper investigates the utilization of digital game in secondary mathematics classrooms. The participants in the study were mathematics teachers from upper secondary schools in Croatia. Using the questionnaire, we collected data on their utilization of digital games and factors that influence their decision to use/not use digital games in the classroom.

Keywords: digital games, mathematics teacher, students, learning, technology

Istraživanje faktora koji utječu na korištenje digitalnih igara u nastavi matematike

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Sažetak. Danas učitelji imaju velike poteškoće u motiviranju učenika i uključivanju učenika u nastavu matematike. Dobar način kojim se može povećati njihovo sudjelovanje je svrhovitom upotrebom igara. Matematičke igre postale su zanimljivo područje u edukacijskim istraživanjima jer se pokazalo da igre razvijaju zanimanje za učenje matematike iščekivanjem natjecanja, izazova i zabave. Igre mogu funkcionirati kao individualne aktivnosti ili kao moćan alat za poučavanje matematičkog sadržaja tijekom nekoliko sati. Da bi igra bila učinkovita, ona se mora uskladiti s ishodom učenja i ne smije biti natjecateljska u svim svojim aspektima. Ponekad igra može zahtijevati zajednički rad učenika na rješavanju problema, dok u drugim situacijama igra može natjerati učenike da se natječu jedni s drugima kako bi postigli osobni rekord. Studija predstavljena u ovom radu istražuje upotrebu digitalnih igara učionica u nastavi matematike u srednjim školama. Sudionici studije bili su srednjoškolski nastavnici matematike iz različitih dijelova Hrvatske. Istraživanje je povedeno putem upitnika kojim smo prikupili podatke rabe li digitalne igre u nastavi matematike i koji faktori utječu na njihovu odluku da koriste ili ne koriste digitalne igre u učionici.

Ključne riječi: digitalne igre, nastavnik matematike, učenici, nastava, tehnologija

Didactic manipulatives in primary mathematics education in Croatia

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Abstract. Didactic materials can be used to introduce abstract mathematical concepts. Various studies found positive effects of using manipulatives in mathematics teaching and learning. Teachers' knowledge and attitudes towards manipulatives influence the profound and helpful implementation of manipulatives in the didactic process. There is a variety of ready-made and self-made manipulatives that can be utilized in mathematics education. Following that perspective, we questioned what role manipulatives play in Croatian primary education. For that purpose, we conducted research with teachers on the use of didactic materials in mathematics classroom teaching. Results showed that teachers use all sorts of manipulatives. They acknowledged the benefits of using manipulatives and most of them claimed they used manipulatives often in the didactic process. However, some answers indicate the lack of knowledge about the characteristics of manipulatives and the implementation of manipulatives in the didactic process.

Keywords: didactic manipulatives, mathematics teacher, primary mathematics education

Didaktički materijali u razrednoj nastavi matematike u Hrvatskoj

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Sažetak. Didaktički materijali mogu poslužiti za uvođenje apstraktnih matematičkih pojmova. Različita istraživanja su pokazala pozitivne učinke korištenja didaktičkih materijala u poučavanju i učenju matematike. Znanje i stavovi učitelja o didaktičkim materijalima utječu na smislenu i korisnu uporabu materijala u nastavi matematike. Postoje mnogi gotovi materijali, dok neke materijale u svrhu korištenja u matematičkom obrazovanju učitelji i samostalno izrađuju. Uzimajući to u obzir, istražili smo koja je uloga didaktičkih materijala u razrednoj nastavi u Hrvatskoj. U tu svrhu, proveli smo istraživanje s učiteljima o uporabi didaktičkih materijala u razrednoj nastavi matematike. Rezultati su pokazali da učitelji koriste različite vrste didaktičkih materijala. Istaknuli su prednosti korištenja didaktičkih materijala te naveli da ih koriste često u nastavi matematike. Ipak, odgovori nekih ispitanika ukazuju na nepoznavanje karakteristika didaktičkog materijala te načina uključivanja didaktičkih materijala u nastavni proces.

Ključne riječi: didaktički materijali, razredna nastava matematike, učitelj matematike

Attitudes towards online learning among students of Faculty of Civil Engineering and Architecture Osijek

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Abstract. This paper presents an analysis of online course in *Mathematics for Engineers II* at the Faculty of Civil Engineering and Architecture Osijek. Due to a pandemic caused by the coronavirus SARS-CoV-2, after only two weeks of regular classes, it was necessary to switch to online performance which lasted till the end of the semester, without any prior preparations. Such work brought a lot of new challenges and in the end, it was important to analyze the opinions of students as the main stakeholders in this process.

The paper presents a quantitative and qualitative analysis of data collected by online survey questionnaire among students. It will be shown that there are no differences in attitudes towards online classes regarding gender and type of finished secondary school. However, it has been shown that students who have passed this course are more satisfied with their personal engagement and work on the e-course as well as what they have learned in online course. Furthermore, the paper analyzes the students' technical preconditions for participating in distance learning, satisfaction with the communication and collaboration among students and teacher and among students themselves as well as students' satisfaction with the choice of tools and activities used in the online course.

Keywords: online learning, mathematics, survey questionnaire, attitudes, undergraduate students

Stavovi studenata Građevinskog i arhitektonskog fakulteta Osijek o provedenoj online nastavi

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Sažetak. U ovom radu je predstavljena analiza provedenog online kolegija *Matematika za inženjere II* na Građevinskom i arhitektonskom fakultetu Osijek. Uslijed pandemije uzrokovane korona virusom SARS-CoV-2 nakon samo dva tjedna redovne nastave moralo se u potpunosti prijeći na online izvedbu koja je potrajala do kraja semestra, a bez ikakvih prethodnih priprema. Takav rad je donio puno novih izazova te je na kraju bilo bitno analizirati mišljenje studenata kao glavnih dionika tog procesa.

U radu će se predstaviti kvantitativna i kvalitativna analiza podataka prikupljenih online anketnim upitnikom među studentima. Pokazat će se da ne postoje razlike u stavovima prema izvedenoj online nastavi s obzirom na spol i prethodno završenu srednju školu. Međutim, pokazano je da su studenti koji su položili taj kolegij zadovoljniji osobnim angažmanom i radom na e-kolegiju kao i naučenim u online nastavi. Nadalje, u radu su analizirane tehničke mogućnosti studenata za praćenje nastave na daljinu, zadovoljstvo ostvarenom komunikacijom i kolaboracijom studenata s nastavnikom i studenata međusobno te zadovoljstvo izborom alata i aktivnosti korištenih u online kolegiju.

Ključne riječi: online nastava, matematika, anketni upitnik, stavovi, studenti

Is it mathematics? Available modelling skills of prospective mathematics teachers

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Abstract. For some time now, world curricula have been prescribing the integration of mathematical modelling into teaching practices, and this is also the case in the Croatian educational system. However, as teachers of prospective teachers of mathematics we experience that tasks considered as modelling tasks in high school textbooks are predominantly of the “application type”. In these tasks function models are already offered and the only thing required from students is the calculation using the given formula, without questioning the dependencies of different variables and properties of functions describing these dependencies (to justify the choice of the function model on the basis of some arguments, to select a function or even to recognize the need for a new one). Our hypothesis is that between so-called “open modelling tasks” often advocated by curriculum documents and, on the other side, “application tasks” just mentioned, there are tasks “in between” that still challenge students’ modelling skills and that can be seen as a basis needed in work with more complex tasks. Still, in implementation of such tasks in our teaching practice, we have noticed that students take a superficial approach regarding modelling, e.g. they often miss naming variables, neglect the use of units or the interpretation of results.

In this paper, we focus on the *skills* required of the students to engage in the modelling process, even on the basic level, and the *conditions and constraints* regarding students’ work with selected modelling tasks which we observed during a modelling workshop. In the workshop, we have involved a cohort of students, prospective teachers of mathematics, and based our analysis of their work on the theoretical frameworks provided by Anthropological Theory of the Didactic and Realistic Mathematics Education.

Keywords: mathematical modelling, preservice teachers, teacher education, type of tasks, functions as models

Je li to matematika? Dostupne vještine modeliranja budućih nastavnika matematike

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Sažetak. Svjetski kurikulumi već dulje vrijeme propisuju integraciju matematičkog modeliranja u nastavnu praksu, što je slučaj i u hrvatskom obrazovnom sustavu. Međutim, kao nastavnici studentima koji su budući nastavnici matematike, doživljavamo da su zadaci koji se u srednjoškolskim udžbenicima smatraju zadacima modeliranja pretežno „tipa primjene“. U tim su zadacima već ponuđeni funkcijski modeli i jedino što se od učenika traži je izračunavanje pomoću poznate formule, bez propitivanja ovisnosti različitih varijabli i svojstava funkcija koje opisuju ovisnosti (da bi se opravdao izbor modela funkcije temeljem nekih argumenata, učinio izbor funkcije kao modela ili da bi se prepoznala potreba za nekom novom funkcijom). Naša je pretpostavka da između takozvanih „zadataka otvorenog modeliranja“ koji se često zagovaraju u kurikularnim dokumentima i, s druge strane, upravo spomenutih „zadataka primjene“ postoje zadaci „između“ koji još uvijek propituju vještine studenata u modeliranju i koji se mogu smatrati osnovom za rad sa složenijim zadacima. Ipak, pri provedbi takvih zadataka u našoj nastavnoj praksi, primijetili smo da studenti imaju površan pristup modeliranju, npr. da često izostavljaju imenovanje varijabli, zanemaruju upotrebu jedinica kao i interpretaciju rezultata.

U ovom radu usredotočujemo se na *vještine* potrebne studentima za sudjelovanje u procesu modeliranja, čak i na osnovnoj razini, te na *uvjete i ograničenja* u vezi s radom studenata s odabranim zadacima modeliranja, a koje smo primijetili tijekom provedene radionice modeliranja. U radionicu je bila uključena cijela generacija studenata, budućih nastavnika matematike, a analizu njihovog rada

temeljimo na teorijskim okvirima koje pružaju Antropološka teorija didaktike (ATD) i Realistično matematičko obrazovanje (RME).

Ključne riječi: matematičko modeliranje, budući nastavnici matematike, obrazovanje nastavnika, tip zadatka, funkcije kao modeli

“Concurrent” method of teaching multiplication and division

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Abstract. This paper proposes the “concurrent” method to teaching multiplication and division in elementary school. The term “concurrent” implies the basic characteristic of the method which is to simultaneously introduce both arithmetic operations. The concurrent method supports making connections between arithmetic operations and ease memorization of the table of multiplication and division. The theoretical rationale for the concurrent method is based on the principle of connections as a basic principle of mathematics education. The method was implemented in an innovative textbook for the 2nd grade of elementary school. This paper aims to describe the method, contrasting it with the traditional method. Guidelines for implementing the concurrent method are provided and a model of this method is presented.

Keywords: teaching method, curriculum design, multiplication, division, textbook

„Simultani“ metod podučavanja množenja i deljenja

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Apstrakt. U ovom radu predlažemo „simultanu“ metodu podučavanja množenja i deljenja u osnovnoj školi. Izraz „simultano“ podrazumeva osnovnu karakteristiku metode koja je istovremeno uvođenje obe aritmetičke operacije. Simultana metoda podržava uspostavljanje veza između aritmetičkih operacija i olakšava pamćenje tabele množenja i deljenja. Teorijsko obrazloženje simultane metode zasniva se na principu veza kao osnovnom principu matematičkog obrazovanja. Metoda je primenjena u inovativnom udžbeniku za 2. razred osnovne škole. Cilj ovog rada je da opiše metodu, suprotstavljajući je tradicionalnoj metodi. Date su smernice za primenu simulatne metode i predstavljen je model ove metode.

Ključne reči: nastavna metoda, dizajn kurikuluma, množenje, deljenje, udžbenik

The transition of the concept of a polynomial through undergraduate mathematics courses

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Abstract. Although polynomials occupy a prominent place and a significant part of the Croatian high school mathematics curriculum, many first-year undergraduate students of mathematics and mathematics education have to accommodate their knowledge when encountering definitions of the polynomial concept.

This paper focuses on the comparison and mutual influence of various ways of introducing a polynomial and the corresponding notations used through undergraduate mathematics courses. More precisely, it aims to investigate the cognitive transition of the concept of a polynomial using Action-Process-Object-Schema (APOS) Theory, as well as didactic contracts which deal with or include this concept. Through an analysis of targeted teaching materials and textbooks, we observed obstacles in the polynomial concept acquisition.

The empirical results of questionnaires and interviews conducted among undergraduate students, teaching assistants, and some professors at the Department of Mathematics, University of Split, were analyzed and discussed. The data indicate a strong connection between the coherence of (generic) student's scheme of the polynomial and relevant didactic contracts.

Keywords: University mathematics education, polynomial, APOS Theory, didactic contract, epistemological and didactic obstacles

Tranzicija pojma polinoma kroz kolegije matematičkih preddiplomskih studija

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Sažetak. Iako polinomi zauzimaju važno mjesto i značajan dio hrvatskog srednjoškolskog kurikulumu nastavnog predmeta Matematike, studenti matematičkih preddiplomskih studija (preddiplomski studiji Matematike, Matematike i fizike, Matematike i informatike) već na prvoj godini studija svoja znanja moraju akomodirati pri susretu s definicijama pojma polinoma.

Ovaj članak je usmjeren na usporedbu i međusobni utjecaj različitih načina uvođenja pojma polinoma i pripadajućih oznaka koje se koriste kroz matematičke kolegije ovih preddiplomskih studija. Preciznije, cilj rada je istražiti kognitivnu tranziciju pojma polinoma koristeći Akcija-Proces-Objekt-Shema (APOS) teoriju, kao i didaktičke ugovore koji se bave polinomima ili ih uključuju.

Analizom odgovarajućih nastavnih materijala i udžbenika, uočene su prepreke pri usvajanju koncepta polinoma. Diskutirani su empirijski rezultati upitnika te intervjua provedenih među studentima, asistentima i nekim nastavnicima Odjela za matematiku Prirodoslovno-matematičkog fakulteta Sveučilišta u Splitu. Podaci ukazuju na snažnu vezu između koherentnosti sheme polinoma (generičkog) studenta i relevantnih didaktičkih ugovora.

Ključne riječi: Sveučilišno matematičko obrazovanje, polinomi, APOS teorija, didaktički ugovor, epistemološke i didaktičke prepreke

Open vs. closed tasks in mathematics

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Abstract. By frontally introducing curricular reform “Škola za Život” and making changes in the mathematics curriculum, the focus is placed on research – based teaching and implementing open tasks. There is no doubt that some obstacles would emerge in teaching and learning mathematics when introducing changes. A reason why those obstacles surfaced may be found in teaching practice, practised until now. So called “traditional teaching”, was fixed on standard, closed tasks and procedure drill. The aim of this paper is to investigate how equipped students are for solving open tasks in comparison to closed tasks and how successful they are in both. Two different worksheets were presented to each, students of fourth and fifth grade of elementary school. The first worksheet investigated students’ knowledge of the area by open tasks and the second one by closed tasks. In the results, their achievements are compared.

Keywords: open tasks, closed tasks, research – based teaching, traditional teaching, area

Otvoreni vs. zatvoreni zadaci u nastavi matematike

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Sažetak. Frontalnim uvođenjem Škole za život te promjenom predmetnog kurikulumu stavljen je naglasak na istraživačku nastavu matematike te na uvođenje otvorenih zadataka. Nedvojbeno je da se u takvoj nastavi javljaju određeni problemi pri poučavanju i ostvarivanju ishoda. Smatra se kako je tome doprinijela dosadašnja tzv. tradicionalna nastava usmjerena na standardni, zatvoreni tip zadatka i na uvježbavanje procedura. Cilj je našeg istraživanja bio provjeriti koliko su učenici spremni za rješavanje otvorenih zadataka u usporedbi sa zatvorenim zadacima te koliko su uspješni pri rješavanju obje vrste zadataka. Učenici 4. i 5. razreda osnovne škole rješavali su dva različita radna listića. Prvim radnim listićem i otvorenim tipom zadataka provjeravano je znanje o površinama. Drugim je radnim listićem isto provjeravano znanje o površinama, ali zatvorenim tipom zadataka. Usporedba učeničkih uspjeha prikazana je u rezultatima istraživanja.

Ključne riječi: otvoreni zadatci, zatvoreni zadatci, istraživačka nastava, tradicionalna nastava, površina

“Multiplication table, at last!” - Teaching of multiplication in Croatia

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Abstract. The content analysis of Croatian mathematics textbooks for primary education indicates that in Croatia the initial learning of concepts and skills related to multiplication relies heavily on practice. At the same time, students are not provided with greater support in the form of various strategies and representations that can facilitate their learning. Therefore, it is to be expected that students will largely experience multiplication and the multiplication table as something that simply needs to be learned by heart (Baković, Trupčević, & Valent, 2019).

Textbooks as educational artifacts approved by state bodies responsible for education policy form an important element of the subject curriculum and have a significant impact on teaching practice (Haggarty & Pepin, 2002; Glasnović Gracin & Jukić Matić, 2016). But as learning and teaching represent a complex system, to get a fuller picture it is also necessary to look at how the teaching of multiplication looks in practice itself.

For the purposes of the research, it was arranged with six teachers of 2nd grade of primary school from Zagreb and Petrinja, two cities from central Croatia, to observe their teaching of multiplication in their classes. For the analysis of observed classes, a framework from Charalambous, Delaney, Hsu, and Mesa (2010) was adapted, as it was done in the previous textbook analysis. The research plan envisaged observation of three teaching hours per teacher, but as classes ceased to be held live in early March due to the Covid-19 pandemic, only one teaching hour per teacher was observed. The data obtained in this way were compared with the data obtained from the analysis of the relevant textbooks used by the teachers.

In this presentation, we present the results of this incomplete research, believing that they can serve as motivation for new research in the near or distant future.

Keywords: classroom observation, concept construction, initial multiplication, textbook analysis

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„Napokon, tablica!“ - poučavanje množenja u Hrvatskoj

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Sažetak. Analiza sadržaja hrvatskih udžbenika matematike za razrednu nastavu naznačuje da se u Hrvatskoj početno učenje koncepata i vještina vezanih uz množenje u velikoj mjeri oslanja na uvježbavanje. Pritom se učenicima ne pruža veća podrška u vidu različitih strategija i reprezentacija koje im mogu olakšati to učenje. Stoga je za očekivati da će učenici u velikoj mjeri doživjeti množenje i tablicu množenja kao nešto što se jednostavno treba naučiti napamet (Baković, Trupčević, & Valent, 2019).

Udžbenici kao obrazovni artefakti odobreni od državnih tijela nadležnih za obrazovnu politiku čine bitan element kurikuluma predmeta te imaju značajan utjecaj na nastavnu praksu (Haggarty & Pepin, 2002; Glasnović Gracin & Jukić Matić, 2016). No, kako su učenje i poučavanje kompleksan sustav, da bismo dobili potpuniju sliku potrebno je i pogledati kako poučavanje množenja izgleda u samoj nastavnoj praksi.

Za potrebe istraživanja dogovoreno je opažanje nastave množenja u 2.r. osnovnih škola sa šest učiteljica iz Zagreba i Petrinje, dva grada iz središnje Hrvatske. Za analizu opažanih sati prilagođen je okvir od Charalambous, Delaney, Hsu i Mesa (2010), kakav je korišten i u prethodnoj analizi udžbenika. U planu istraživanja je bilo predviđeno opažanje tri nastavna sata po učitelju, no, kako se početkom ožujka zbog pandemije bolesti Covid-19 nastava prestala održavati uživo, obavljeno je opažanje samo jednog nastavnog sata svake učiteljice. Podaci dobiveni na taj način uspoređeni su s podacima dobivenim analizom odgovarajućih udžbenika koje su nastavnici koristili.

U ovom izlaganju prezentiramo rezultate ovog nepotpunog istraživanja, uz uvjerenje da oni mogu poslužiti kao motivacija za novo istraživanje u bližoj ili daljoj budućnosti.

Ključne riječi: opažanje nastave, konstrukcija koncepta, početno množenje, analiza udžbenika

What do mathematics teachers believe about professional development, educational research, and teaching?

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Abstract. Bringing significant changes to teaching mathematics requires focus on teachers because they are the most influential source of change that can make a difference. Teachers' instructional practice gets influenced by their knowledge and beliefs. Scholars have long argued that we have to invest in effective professional development if we want to empower teachers to engage in meaningful and sustained improvement. A few studies indicate that using research results can improve teaching practice, in particular the teacher-researcher concept. Teachers will change their attitudes and beliefs if they gain evidence of progress in students' learning. The purpose of this study is to develop a questionnaire that will provide some base-line data about teachers' beliefs on professional development, educational research, and nature and teaching mathematics. The questionnaire was conducted online with the elementary school mathematics teachers in Koprivnicko-Krizevacka county, Croatia. The analysis has shown that mathematics teachers attend various professional development programs but rarely use knowledge or materials obtained during these programs. The majority of the participants have never conducted or participated in educational research. Yet, teachers say they are using scientific results to improve their teaching practice, and at the same time, they rarely read scientific papers.

Keywords: mathematics teachers, beliefs, professional development, educational research, questionnaire

Što vjeruju nastavnici matematike o profesionalnom usavršavanju, istraživanju u obrazovanju i poučavanju?

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Sažetak. Donošenje značajnih promjena u poučavanju matematike zahtijeva usredotočenost na nastavnike jer su oni najutjecajniji izvor promjena. Na nastavnu praksu utječu znanje i uvjerenja nastavnika. Znanstvenici već dugo tvrde da moramo ulagati u učinkovita profesionalna usavršavanja ako želimo osnažiti nastavnike da se uključe u značajna i trajna poboljšanja. Nekoliko studija ukazuje na to da korištenje rezultata istraživanja može poboljšati nastavnu praksu, posebno koncept nastavnik-istraživač. Nastavnici će promijeniti svoje stavove i uvjerenja ako vide dokaze o napretku u učenju svojih učenika. Svrha ovog istraživanja je razviti upitnik koji će pružiti osnovne podatke o vjerovanjima nastavnika o profesionalnom razvoju, istraživanjima u obrazovanju, te prirodi i poučavanju matematike. Upitnik je proveden putem interneta za osnovnoškolske nastavnike matematike u Koprivničko-križevačkoj županiji, Hrvatska. Analiza je pokazala da učitelji matematike pohađaju razne programe profesionalnog razvoja, ali rijetko koriste znanje ili materijale stečene tijekom tih programa. Većina sudionika nikada nije provodila niti sudjelovala u istraživanjima u obrazovanju. Pa ipak, nastavnici kažu da koriste informacije temeljene na istraživanjima kako bi poboljšali svoju nastavnu praksu, a istodobno rijetko čitaju znanstvene radove.

Ključne riječi: učitelji matematike, vjerovanja, profesionalno usavršavanje, istraživanje u obrazovanju, upitnik

Mathduel, a math game for high school

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Abstract. Playing games is an essential activity for every child, the imperative way children learn. So, it is no surprise that a lot of research explores the influence games have on students learning and achievements. Research shows that the use of educational games in the classroom has a positive impact on learning, motivation, students’ cognitive abilities, and students’ achievements. With that in mind, it is interesting to see that using educational games as a teaching strategy in Croatian high schools is minimal. MathDuel is a mathematical game designed as a teaching and learning aid in high school math education. Playing MathDuel helps students develop better conceptual knowledge by connecting different mathematical concepts in creative and resourceful ways. With this math game, we want to enrich teaching and learning mathematics.

This paper will present MathDuel: the idea, constructing process, rules, and gameplay.

Keywords: game, mathematics, conceptual knowledge, math teaching, students’ achievements

Matoboj, matematička igra za srednju školu

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Sažetak. Igranje je osnovna aktivnost svakog djeteta, najvažniji način na koji djeca uče. Stoga ne iznenađuje što mnoga istraživanja istražuju utjecaj igara na učenje i postignuća učenika. Istraživanja pokazuju da korištenje obrazovnih igara u učionici pozitivno utječe na učenje, motivaciju, kognitivne sposobnosti i postignuća učenika. Imajući to na umu, zanimljivo je vidjeti da je korištenje obrazovnih igara kao strategije poučavanja u hrvatskim srednjim školama minimalno. Matoboj je matematička igra zamišljena kao pomoć u nastavi i učenju matematike u srednjoškolskom obrazovanju. Igranje Matoboja pomaže studentima razviti bolje konceptualno znanje povezivanjem različitih matematičkih pojmova na kreativan i domišljat način. Ovom matematičkom igrom želimo obogatiti nastavu i učenje matematike.

U ovom radu predstavljamo Matoboj: ideju, nastanak, pravila i kako igrati igru.

Ključne riječi: igra, matematika, konceptualno znanje, poučavanje matematike, učenička postignuća