

**The 7th International Scientific Colloquium
MATHEMATICS AND CHILDREN
founded by Margita Pavleković**

BOOK OF ABSTRACTS

Editors:

**Zdenka Kolar-Begović
Ružica Kolar-Šuper
Ljerka Jukić Matić**

Croatia, Osijek, May 24-25, 2019

**Josip Juraj Strossmayer University of Osijek
Faculty of Education and Department of Mathematics**

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Contents

Plenary lectures

Tatjana Hodnik	
<i>Mathematical problem solving: prospects and reality</i>	1
Ana Kuzle	
<i>What can we learn from students' drawings?</i>	
<i>Visual research in mathematics education</i>	7
Sanja Rukavina	
<i>Preservice mathematics teachers and teacher research.</i>	9

Contributed talks

Tihana Baković, Goran Trupčević, Andja Valent	
<i>Treatment of initial multiplication in textbooks from Croatia and Singapore.</i>	11
Tarik Baždalić, Maid Omerović	
<i>Catenary</i>	13
Maja Cindrić, Irena Mišurac Zorica, Josipa Jurić	
<i>From "calculation in mind" till "mental calculation"</i>	15
Edith Debrenti, Balázs Vértesy	
<i>Mathematical problem solving in practice.</i>	17
Karolina Dobi Barišić	
<i>Teaching with the use of ICT - how teachers perceive their own knowledge?</i>	19
Ivana Đurđević Babić, Dajana Sabolić	
<i>Mining students' viewpoints about programming in primary education</i>	23
Vjekoslav Galzina, Roberto Lujic	
<i>Simple mathematical model of Cyber-Physical System</i>	27
Amanda Glavaš, Azra Staščik, Ljerka Jukić Matić	
<i>What types of knowledge do mathematics textbooks promote</i>	29

Željko Gregorović, Ana Katalenić <i>Primary school teachers' (mis)understandings of equality and the equals sign</i>	31
Matea Gusić <i>Investigating adaptive reasoning and strategic competence in Croatian mathematics education: The example of quadratic function.</i>	33
Anes Z. Hadžiomerović, Amila Osmić <i>Poincaré's model of hyperbolic geometry</i>	35
Marija Jakuš, Lucija Žignić <i>Generating question for Moodle base.</i>	37
Ljerka Jukić Matić, Dubravka Glasnović Gracin <i>The influence of teacher guides on classroom practice</i>	39
Josipa Jurić, Irena Mišurac Zorica, Maja Cindrić <i>Student competence for solving logical tasks</i>	41
Zdenka Kolar-Begović, Ružica Kolar-Šuper, Ivana Đurđević Babić, Diana Moslavac Bičvić <i>Pre-service teachers' prior knowledge related to measurement</i>	43
Nikolina Kovačević <i>The use of mental geometry in the development of the geometric concept of rotation. . . 45</i>	
Zoltán Kovács, Eszter Kónya <i>How do novices and experts approach an open problem?.</i>	47
Ljiljanka Kvesić, Slavica Brkić <i>Mathematical abilities of pre-school children</i>	49
Josipa Matotek <i>Computer-based assessments in mathematics at the higher education level</i>	51
Željka Milin Šipuš, Aleksandra Čižmešija, Ana Katalenić <i>Redesigning a contextual textbook task with an exponential-type function using a posteriori analysis of the prospective mathematics teachers' work</i>	53
Ana Mirković Moguš <i>The role of online applications as a tool of support in mathematics education.</i>	55
Emil Molnár, István Prok, Jenő Szirmai <i>From a nice tiling to theory and applications.</i>	57
Sanela Nesimović, Karmelita Pjanić <i>Teachers' opinions on geometric contents in the curriculum for the lower grades of primary school</i>	61

Sanela Nesimović, Karmelita Pjanić <i>Geometric thinking of primary school pupils.</i>	63
Ahmed Palić, Maid Omerović, Edisa Korda <i>The principle of mathematical induction and Peano's axioms, their definition and application through the prism of the methods of mathematics and mathematical competences of the mathematics teachers</i>	67
Sead Rešić, Fatih Destović, Nermin Hodžić <i>Midlines of a quadrilateral.</i>	69
Sead Rešić, Ahmed Palić, Edisa Korda <i>The influence of inclusion on the conative and cognitive characteristics of children in mathematics teaching.</i>	71
Sead Rešić, Fatih Destović, Alma Šehanović, Amila Osmić <i>Problems and problem situation at the teaching topic example "Number divisibility and applications"</i>	73
Ksenija Romstein <i>Technology use in early childhood.</i>	75
Eleonóra Stettner <i>Relationship between the Poly-Universe Game and mathematics education</i>	79

ABSTRACTS

Mathematical problem solving: prospects and reality

Tatjana Hodnik

Faculty of Education, University of Ljubljana, Slovenia

Abstract. Problem solving is a leading mathematical activity which stimulates mathematical thinking. From the theoretical point of view this activity is very complex due to different issues which describe what problem solving is and what is its role in the process of teaching and learning mathematics. Our interest regarding these issues mainly focuses on the following: what are basic characteristic of a mathematical problem, the nature (conceptual, procedural) and the role of representation (interplay between internal and external) of a mathematical problem, mental schemas for problem solving, heuristics as principles, methods and (cognitive) tools for solving problems, types of generalisations and reasoning (abductive, narrative, naïve, arithmetic, algebraic), problem solving as a challenging activity for mathematically gifted students and role of teacher's guiding of problem solving as a way of implementing students' problem solving in the classroom. Studies we carried out on problem solving in Slovenian context, which we believe are relevant also to other country contexts, include different groups of students at all levels (from elementary school, secondary school, prospective teachers) are presented aiming to encourage our thinking about prospects of problem solving in the process of teaching and learning mathematics in relation to reality of mathematics classrooms. We believe that problem solving has different roles and benefits for the learners (beside the activity of problem solving itself), namely (we are presenting those we have investigated): students can through problem solving demonstrate their pre-knowledge about a certain mathematical concept [1], problem solving activities provoke students' creativity [2], problem solving is challenging for gifted students, because they are interested in complex problems, which lead to different solutions, problems encourage them to take decisions, include uncertainty...[3], problems

solving enables diagnosis of students' misconceptions about mathematical concepts [4], an analysis of students' problem solving activity provides us knowledge about students' mental schemas for problem solving [5]. Research has proven and it was confirmed many times that there is a great potential of (mathematical) problem solving for the learners but the reality in terms of teaching practice, external examinations, teaching material, mathematics curriculum seems not to be in tune with the research findings. We are going to present some possible reasons why we are facing this gap and give some ideas for improvement the situation (arguing that problem solving must take place in the mathematics classroom). Among others we claim that going back to heuristics, teach and guide students' problems solving (meaning developing systematic approach to problems solving), as a complement to constructivist ideas (where students are encouraged to develop problem solving strategies themselves, using the knowledge they possess and developing new) might be a good 'start' to put forward problem solving activities in the classroom.

Keywords: problem solving, inductive reasoning, heuristics, mental schema, teaching practice, research on problem solving

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Reševanje matematičnih problemov: obeti in realnost

Tatjana Hodnik

Pedagoška fakulteta, Univerze v Ljubljani, Slovenija

Povzetek. Reševanje (matematičnih) problemov je vodilna aktivnost (v matematiki), ki spodbuja (matematično) mišljenje in usmerja razvoj (matematike). Če na to aktivnost pogledamo s teoretičnega vidika, predvsem v povezavi z izobraževanjem, ugotovimo, da imamo opraviti z zelo kompleksno situacijo. Reševanje problemov v tem kontekstu lahko preučujemo z različnih vidikov, kot a primer (omenili bomo zgolj nekatere): katere so bistvene karakteristike matematičnih problemov, katere vrste matematičnih problemov poznamo, kakšen je pomen reprezentacij oz. na kakšen način reševalec vzpostavlja povezave povezav med zunanjimi in notranjimi reprezentacijami, kakšno vlogo imajo miselne sheme za reševanje problemov, kako hevristike kot principi in (kognitivna) orodja za reševanje problemov pomagajo reševalcu pri soočenju s problemi, na kakšne načine lahko posplošujemo rešitev, kjer nas zanima predvsem induktivno sklepanje, ki ga delimo v glavnem na abduktivno, narativno, naivno, aritmetično in algebraično posploševanje. Z vidika vloge reševanja problemov pri pouku pa je po našem prepričanju ključni izpostaviti vlogo učitelja, ki na osnovi znanja in premišljenega vodenja vpeljuje reševanje problemov v pouk matematike. Veliko različnih raziskav je bilo opravljenih na področju preučevanja reševanja problemov v Sloveniji, ki so za razumevanje te problematike aktualne tudi širše, izven slovenskega prostora. Prikazane študije, s katerimi želimo vzpodbuditi razmišljanja o obetih reševanja problemov pri pouku, vključujejo različne skupine preučevancev (od predšolskih otrok do študentov na univerzi) ter obravnavajo različne, zgoraj omenjene, vidike reševanja problemov. Verjamemo namreč, da ima reševanje problemov za učence, poleg pomena dejavnosti kot take, tudi druge pomene in sicer (predstavili bomo tiste, ki smo jih preučevali v naših raziskavah): učenci z reševanjem problemov izkažejo predznanje o izbranem matematičnem pojmu [1], reševanje problemov v učencih izzove kreativnost [2], reševanje problemov, predvsem kompleksnih, predstavlja

izziv nadarjenim učence, ker jim omogočajo različne rešitve, jih spodbujajo k sprejemanju odločitev, vključujejo negotovost...[3], na osnovi učenčevega reševanja problemov lahko ugotovimo njegove napačne predstave o izbranem pojmu [4], analiza reševalčevih strategij omogoča pridobivanje znanja o miselnih shemah za reševanje problemov [5]. Raziskave vedno znova pokažejo, da ima reševanje (matematičnih) problemov veliko potencialov za učenje, a v realnosti, ki jo na tem mestu razumemo kot poučevanje v razredu, različni konteksti preverjanja znanja (nacionalni in mednarodni), učbeniška gradiva ter učni načrt, pa s trendi raziskav praktično niso uglašeni. Izpostavili bomo nekatere možne razloge, zakaj do teh razhajanj pride in poskušali podati nekatere rešitve, pri čemer nas vodi stališče, da je potrebno vztrajati pri ideji implementiranja reševanja problemov v pouk matematike na vseh stopnjah izobraževanja. Med drugim bomo zagovarjali, da poučevanje hevristik in vodenje učenca pri reševanju problemov ali drugače, sistematičen pristop k reševanju problemov kot komplement konstruktivističnemu pristopu, lahko predstavlja dober ‘začetek’ (Polya je s to idejo pričel pred več kot 70-imi leti) za napredovanje pri udejanjanju reševanja problemov pri pouku matematike.

Keywords: reševanje problemov, induktivno sklepanje, hevristike, miselna shema, poučevanje, raziskave

What can we learn from students' drawings? Visual research in mathematics education

Ana Kuzle

Department of Primary Mathematics Education, University of Potsdam, Potsdam, Germany

Abstract. Over the last twenty years, visual research methods became somewhat mainstream across social science research. Visual research methods incorporate some kind of imagery (e.g., drawings, photographs, videos) into the research process, which then constitute data used to communicate the research results. The roots of visual research stem from social sciences with a focus on the psychological stance of describing children's drawings in terms of developmental sequences. Recently, however, the focus shifted onto using drawings as expressions of meaning, understanding, and affect in (mathematics) education. Especially, participant-produced drawings allow a constructive process of thinking in action, rather than seeing drawings as simple representations of the participant's world. In this plenary lecture, I discuss the possibilities of using drawings in mathematics education research. Concretely, I focus on participant-produced drawings as a window into students' perceptions of geometry classroom learning milieu from a cognitive, social, and emotional perspective.

Keywords: drawings, fundamental ideas, classroom social climate, emotional atmosphere, visual research

Was können wir aus Schülerzeichnungen lernen? Visuelle Forschung in Mathematikdidaktik

Ana Kuzle

Department Lehrerbildung, Grundschulpädagogik Mathematik, Universität Potsdam,
Potsdam, Deutschland

Zusammenfassung. In den letzten zwanzig Jahren haben sich Methoden der visuellen Forschung in den Sozialwissenschaften durchgesetzt. In der visuellen Forschung gibt es verschiedene Möglichkeiten zum Erfassen mentaler Vorstellungen. Dazu zählen zum Beispiel Fotografien, Videoaufnahmen oder Zeichnungen. Die Wurzeln der visuellen Forschung liegen im Schwerpunkt auf der psychologischen Haltung. In letzter Zeit verlagerte sich in Mathematikdidaktik der Fokus jedoch auf die Verwendung von Zeichnungen als Ausdruck von u.a. Schülervorstellungen zum Mathematikunterricht und kognitiven (Annahmen und Überzeugungen), affektiven (Gefühle und Emotionen) und behavioralen (Verhaltensweisen) Einstellungen der Schüler. Insbesondere „participant-produced drawings“ ermöglichen einen ko-konstruktiven Prozess des Denkens in Aktion, anstatt Zeichnungen als einfache Darstellungen der Teilnehmerwirklichkeit zu betrachten. In diesem Paper diskutiere ich die zahlreichen Verwendungsmöglichkeiten von Zeichnungen in mathematikdidaktischer Forschung. Insbesondere fokussiere ich Schülerzeichnungen um einen non-verbalen Einblick in die Wahrnehmungen der Schüler über das geometrische Lernklima aus kognitiver, sozialer und emotionaler Perspektive zu erlangen.

Schlagwörter: Zeichnungen, fundamentale Ideen, Unterrichtsklima, emotionales Klima, visuelle Forschung

Preservice mathematics teachers and teacher research

Sanja Rukavina

Department of Mathematics, University of Rijeka, Croatia

Abstract. Teacher research is intentional and systematic inquiry carried out by teachers. Mainly, it is recognized as an effective way of improving everyday classroom practice, but it can also be a starting point for a systematic investigation of a wider problem. Furthermore, recent changes in the Croatian Education System recognize the importance of the role of a teacher as an active participant in the changing of the educational process. The School for Life program needs teachers willing to conduct teacher research and to take initiative and responsibility in the shaping of the “school for life”. Which problems can be expected during that process? Are preservice mathematics teachers prepared for that challenge? Which changes in their education are needed? These questions will be discussed based on the findings collected from the students of the graduate teacher training courses at the Department of Mathematics, University of Rijeka.

Keywords: teacher research, preservice teacher, role of the teacher, mathematics teaching, School for Life

Budući nastavnici matematike i istraživanje u nastavi

Sanja Rukavina

Odjel za matematiku, Sveučilište u Rijeci, Hrvatska

Sažetak. Nastavničko istraživanje je akcijsko istraživanje usmjerenotovenstveno na poboljšanje vlastite prakse, a može poslužiti kao polazna točka za sustavno istraživanje većeg problema uočenog u nastavnoj praksi. Nedavne promjene u hrvatskom obrazovnom sustavu prepoznaju značaj uloge nastavnika kao aktivnog sudionika i kreatora obrazovnog procesa. Za uspješnu realizaciju programa Škola za život potrebni su nastavnici koji će provoditi istraživanja u nastavi i preuzeti inicijativu i odgovornost za oblikovanje “škole za život”. Koji se problemi mogu očekivati tijekom tog procesa? Jesu li budući nastavnici matematike spremni za taj izazov? Koje su promjene u njihovom obrazovanju potrebne? O ovim ćemo pitanjima raspravljati na temelju saznanja dobivenih od studenata diplomske studije nastavničkog smjera Odjela za matematiku Sveučilišta u Rijeci.

Ključne riječi: akcijsko istraživanje, budući nastavnici, uloga nastavnika, nastava matematike, Škola za život

Treatment of initial multiplication in textbooks from Croatia and Singapore

Tihana Baković¹, Goran Trupčević¹ and Anda Valent²

¹ Faculty of Teacher Education, University of Zagreb, Petrinja, Croatia

² Zagreb University of Applied Sciences, Croatia

Abstract. In this lecture, we aim to partially characterize the way of teaching mathematics, specifically the initial learning of multiplication in the Croatian education system.

Because teaching is a socio-cultural phenomenon, in describing it, it is necessary to step out of the cultural frame, since only then one can notice some of its attributes that from the inside appear to be self-evident.

For this reason, we compared the treatment of initial multiplication learning, including the multiplications table, in textbooks from Croatia and Singapore. In the analysis of the textbooks we used an adapted framework from Charalambous, Delaney, Hsu and Mesa (2010) that looks at a textbook as an environment for construction of knowledge of a single mathematical concept.

Keywords: mathematics textbooks, analysis of concept construction, initial multiplication

Utjecaj uporabe interaktivnih apleta kreiranih u GeoGebri na konceptualno i proceduralno znanje učenika

Tihana Baković¹, Goran Trupčević¹ i Andja Valent²

¹ Učiteljski fakultet, Sveučilište u Zagrebu, Petrinja, Hrvatska

² Tehničko veleučilište u Zagrebu, Hrvatska

Sažetak. U ovom ćemo radu djelomično okarakterizirati način podučavanja matematike, posebice početnog učenja množenja u hrvatskom obrazovnom sustavu.

Budući je podučavanje socio-kulturalni fenomen, kako bi ga se opisalo, potrebno je iskoračiti iz danog kulturnog okvira, jer tek tada možemo uočiti neke njegove atribute koji se iznutra čine očiglednim.

Zbog toga smo usporedili tretman početnog učenja množenja, uključujući tablicu množenja, u udžbenicima iz Hrvatske i Singapura. U analizi udžbenika koristili smo prilagođeni okvir Charalambous, Delaney, Hsu i Mesa (2010) koji na udžbenik gleda kao na okruženje za konstrukciju znanja o pojedinom matematičkom konceptu.

Ključne riječi: matematički udžbenici, analiza formiranja koncepta, početno množenje

Catenary

Tarik Baždalić¹ and Maid Omerović²

¹ Gymnasium „Muhsin Rizvić“ Kakanj, Bosnia and Herzegovina

² Faculty od Education, University of Travnik, Bosnia and Herzegovina

Abstract. By examining the basic assumptions in the static calculations of some constructions, we conclude that on the basis of the optimum choice of carriers, such constructive systems emerge that have assumed the appearance of graphic functions that were later pronounced in the mathematical analysis as basic functions. A characteristic example of this claim is a static constructive catenary type system. In the theory of the construction of a statically determined carrier, it has been shown that the structural system of the catenary type (or a type of a thread suspended from two points) has in fact a graph of the hyperbolic function: $y = ch x$.

This type of construction is most often used in bridge construction when bridging wide rivers, narrow bays and seas. Although the constructional catenary type system used in engineering is very complex and shift constructions use state-of-the-art construction techniques, it was used in the tribal community when deep valleys and canyons were to be bridged. This is best exemplified by the American Indians who made ropes of plants and animal origin ropes, and then from these ropes and wooden beams they made hanging constructions over deep and narrow valleys and river canyons.

Keywords: catenary, load, axial forces, supple thread, static balance

Lančanica

Tarik Baždalić¹ i Maid Omerović²

¹ Gimnazija „Muhsin Rizvić“ Kakanj, Bosna i Hercegovina

² Edukacijski fakultet- Univerzitet Travnik, Bosna i Hercegovina

Sažetak. Ispitivanjem osnovnih pretpostavki u statičkim proračunima nekih konstrukcija zaključujemo da se na temelju optimalnog izbora nosača pojavljuju takvi konstruktivni sistemi koji su prepostavili pojavu grafičkih funkcija koje su kasnije matematički analizirane kao osnovne funkcije. Karakteristični primjer ove tvrdnje je statički konstruktivni tip sistema lančanice. U teoriji konstrukcije statički određenog nosača, pokazalo se da konstrukcijski sistem tipa lančanice (ili vrsta niti koja je obješena na dvije tačke) zapravo ima graf hiperboličke funkcije: $y = ch x$.

Ova vrsta gradnje najčešće se koristi u izgradnji mostova pri premošćivanju širokih rijeka, uskih uvala i mora. Iako je konstrukcija sistema tipa lančanice koja se koristi u inženjerstvu vrlo složena, a konstrukcije pomaka koriste najsavremenije tehnike gradnje, koristile su se u plemenskoj zajednici kada je trebalo premostiti duboke doline i kanjone. O tome najbolje svjedoče američki Indijanci koji su napravili konopce od biljaka i užad životinjskog porijekla, a zatim su iz tih užadi i drvenih greda izrađivali viseće konstrukcije preko dubokih i uskih dolina i riječnih kanjona.

Ključne riječi: lančanica, opterećenje, aksijalne sile, elastična nit, statička ravnoteža

From “calculation in mind” till “mental calculation”

Maja Cindrić¹, Irena Mišurac Zorica² and Josipa Jurić²

¹ Department of Teacher and Preschool Teacher Education, University of Zadar, Croatia

² Faculty of Philosophy, University of Split, Croatia

Abstract. Through the history of mathematics teaching in the territory of Croatia, there have been many changes, one of them being linked to the approach, attitudes and even the very name of what we call today mental calculation. Mental calculation bridge of conceptual and procedural mathematical knowledge and derives from activities at the very beginning of mathematics learning. Depending on the time and needs of society and the economy, consequences of certain educational reforms and quantitative indicators at the regional and global level attitudes toward mental calculation as part of math teaching have changed. Attitudes are often equated with the use of algorithms in written calculations, which is marginalized within mathematics teaching, either because of the over-utilized use of procedural methods of a written calculation, or because of the emphasis on problem solving mathematical tasks for the development of conceptual knowledge.

This paper will give a historical overview of the approach to a mental calculation by analyzing mostly textbooks for teaching mathematics in the territory of today Republic of Croatia as well as the term used for these purposes.

Keywords: mental calculation, calculation in mind, oral calculation

Od “računa napamet” do “misaonog računanja”

Maja Cindrić¹, Irena Mišurac Zorica² i Josipa Jurić²

¹ Odjel za izobrazbu učitelja i odgojitelja Sveučilišta u Zadru, Hrvatska

² Filozofski fakultet u Splitu, Sveučilište u Splitu, Hrvatska

Sažetak. Kroz povijest nastave matematike na području Hrvatske dolazilo je do mnogih promjena, a jedna od njih veže se i za pristup, stavove, pa i sam naziv onoga što danas zovemo misaonim računom. Misaono računanje spona je konceptualnog i proceduralnog matematičkog znanja, a potječe od aktivnosti u samim početcima učenja matematike. Ovisno o vremenu i potrebama društva i gospodarstva, posljedicama određenih odgojno obrazovnih reformi i kvantitativnim pokazateljima na regionalnoj i globalnoj razini mijenjali su se stavovi prema potrebi računanja kao dijelu nastave matematike. Stavovi su često izjednačavali primjenu algoritama u računanju s misaonim računom, koji je u okviru nastave matematike marginaliziran, bilo zbog prenaglašene upotrebe proceduralnih metoda pisanog računa, bilo zbog nastojanja da se za razvoj konceptualnog znanja naglasak stavi na problemske matematičke zadatke.

Ovaj rad će dati povjesni pregled pristupa misaonom računu analizom većinom udžbeničke literature za nastavu matematike na teritoriju današnje Republike Hrvatske, kao i nazivlju koji je korišten u te svrhe.

Ključne riječi: račun napamet, usmeni račun, misaoni račun, mentalni račun, aritmetika

Mathematical problem solving in practice

Edith Debrenti¹ and Balázs Vértesy²

¹Faculty of Economics and Social Sciences, Partium Christian University, Oradea, Romania

²Institute of Mathematics, University of Debrecen, Hungary

Abstract. In elementary education, in the course of solving word problems, we can use multiple methodologies like visualization, fake perception, reversed way (backward induction), comprehension, elimination, evaluation, integration and scale method.

We have a problem, if there is a goal, a task to be solved, but we do not know the way to achieve it, we are facing an obstacle. The way to overcome the obstacle is the problem solving process. The search for and elaboration of the solution is called problem solving. Problem solving thinking is influenced by several factors: precognition, strategies, metacognitive skills, conviction, attitudes as well as self - management.

Problem solving depends on how you manage to mobilize your existing knowledge, the ability to engage in active analysis, the reflectiveness and the creative attitude.

We surveyed 4th graders (10 years old pupils) and student teachers with a test in which we collected exercises which can be solved with different methodologies. It is an exploratory study, with participants from two countries: Hungary and Romania. We were investigating the participants' flexible thinking, the knowledge and diversity of their problem solving skills as well as their accuracy and the most common problems which occurred in the test.

Keywords: knowledge, problem-solving skills, unusual problems, heuristics, strategies

Matematikai problémamegoldás a gyakorlatban

Edith Debrenti¹ és Balázs Vértesy²

¹Gazdaság és Társadalomtudományi Kar, Partiumi Keresztény Egyetem, Románia

²Institute of Mathematics, University of Debrecen, Hungary

Kivonat. Az elemi oktatásban, a szöveges feladatok megoldása során többféle módszert használhatunk. Ilyenek az ábrázolás módszere, a hamis feltevés módszere (például az ültetéses vagy fejes-lábas feladatok), a fordított út (visszafele következtetés) módszere, az összehasonlítás módszere, a kiküszöbölés módszere, a behelyettesítés módszere, az egységre való visszavezetés és a mérlegelv módszere.

Problémáról akkor beszélünk, ha van egy cél, egy megoldandó feladat, de nem ismerjük az eléréséhez vezető utat, akadályba ütközünk. Az akadály leküzdésének útja a problémamegoldás folyamata, a megoldás keresését és kidolgozását nevezzük problémamegoldásnak. A problémamegoldó gondolkodást több tényező befolyásolja: előismeret, stratégiák, metakognitív képességek, meggyőződés, beállítódás, de szerepe van az önirányításnak is. Egy probléma megoldása attól függ, hogyan sikerül a meglévő ismereteket, az aktív analízisre való képességet, a gondolkodást, az alkotó hozzáállást mozgósítani. Kutatásunkban 4. osztályos tanulókat, valamint tanítóképzős hallgatókat mértünk fel egy teszt alkalmazásával, melyben különböző módszerrel megoldható feladatokat válogattunk össze. A válaszokban a rugalmas gondolkodást, a megoldási módszerek ismeretét, azok változatosságát, a pontosságot figyeltük és a leggyakrabban előforduló hibákat elemezük. Feltáró vizsgálatunkat két országban végeztük el: Magyarországon és Romániában.

Kulcsszavak: ismeret, problémamegoldási képességek, érdekes feladatok,elfedezés, stratégiák

Teaching with the use of ICT - how teachers perceive their own knowledge?

Karolina Dobi Barišić

Faculty of Education, University of Osijek, Croatia

Abstract. The traditional educational system, operating on the basis of one-way content transfer, does not enable the acquisition of competencies needed in the contemporary society of rapid change and ruthless competition. Digital competence is one of eight core competences highlighted by the Education Council of the European Union and also represent the foundation of Croatian education policy. In Croatia, the importance of new technologies integration into education and research systems is highlighted in the Strategy of Education, Science and Technology, adopted in 2014 (Ministarstvo znanosti, obrazovanja i sporta, 2014).

Theoretical framework Technological Pedagogical Content Knowledge (TPACK) describes how teachers understand the interaction between educational technology, pedagogical and content knowledge. This framework represents the mutual influence of the teacher's understanding of educational technology, pedagogical and content knowledge for the purpose of effective application of educational technology in teaching. Survey of Preservice Teachers' Knowledge of Teaching and Technology (SPTKTT) represents the self-assessment of knowledge on the application of technology in teaching and was developed on the basis of the TPACK framework.

Two hypotheses are considered in this research: (H1) There is a difference in the Croatian teachers' self-perception of knowledge on the use of ICT in teaching in relation to pedagogical and content knowledge, (H2) Teachers, who perceive a higher level of technological knowledge, than pedagogical and content knowledge, show greater preparedness for successful integration of technology into education.

Research was conducted in 2015 among Croatian in-service teachers in elementary schools on a sample of N = 266 teachers. Both hypothesis were

confirmed. According to their self-evaluation, Croatian in-service teachers perceive less knowledge about integration of ICT in teaching in relation to pedagogical and content knowledge. Also, Croatian in-service teachers, that perceive higher level of own technological knowledge, show greater readiness for effective ICT application in teaching.

Keywords: ICT, digital competence, TPACK framework, education, educational technology

Poučavanje uz pomoć IKT-a – kako učitelji doživljavaju vlastito znanje?

Karolina Dobi Barišić

Fakultet za odgojne i obrazovne znanosti, Sveučilište u Osijeku, Hrvatska

Sažetak. Tradicionalni obrazovni sustav, koji funkcionira na temelju jednosmjernog prijenosa znanja, ne omogućuje stjecanje kompetencija potrebnih suvremenom društvu brzih promjena i nemilosrdnog natjecanja. Digitalna kompetencija jedna je od osam temeljnih kompetencija istaknuta u obrazovnom vijeću Europske unije te također predstavlja temelj hrvatske obrazovne politike. U Hrvatskoj je važnost integracije novih tehnologija u sustav obrazovanja i istraživanja istaknuta u Strategiji obrazovanja, znanosti i tehnologije, usvojene 2014. godine (Ministarstvo znanosti, obrazovanja i sporta, 2014).

Teorijski okvir Tehnološko pedagoško sadržajna znanja (TPACK) opisuje kako bi nastavnici trebali razumjeti interakciju obrazovne tehnologije, pedagoškog i sadržajnog znanja. Ovaj okvir predstavlja uzajamni utjecaj učiteljevog razumijevanja obrazovne tehnologije, pedagoškog i sadržajnog znanja u svrhu učinkovite primjene obrazovne tehnologije u nastavi. Survey of Pre-service Teachers' Knowledge of Teaching and Technology (SPTKTT) predstavlja samoprocjenu vlastitih znanja o primjeni tehnologije u nastavi i razvijen je na temelju TPACK teorijskog okvira.

U ovom istraživanju se razmatraju dvije hipoteze: (H1) Postoji razlika u samoprocjeni znanja hrvatskih učitelja o korištenju ICT-a u nastavi u odnosu na pedagoško i sadržajno znanje, (H2) Nastavnici, koji posjeduju viši stupanj tehnološkog znanja, od pedagoškog i sadržajnog znanja, pokazuju veću spremnost za uspješnu integraciju tehnologije u obrazovanje.

Istraživanje je provedeno 2015. godine među nastavnicima u osnovnim školama na uzorku od $N = 266$ nastavnika. Obje hipoteze su potvrđene. Samoprocjenom je utvrđeno da hrvatski učitelji posjeduju slabija znanja o integraciji IKT-a u nastavu u odnosu na pedagoška i sadržajna znanja. Također, hrvatski učitelji,

koji posjeduju višu razinu vlastitog tehnološkog znanja, pokazuju veću spremnost za učinkovitu primjenu IKT-a u nastavi.

Ključne riječi: IKT, digitalna kompetencija, TPACK teorijski okvir, obrazovanje, obrazovna tehnologija

Mining students' viewpoints about programming in primary education

Ivana Đurđević Babić and Dajana Sabolić

Faculty of Education, University of Osijek, Croatia

Abstract. Nowadays, it is desirable to acquire computer programming competencies and knowledge in different fields of programming. It is well known that someone's viewpoints and attitudes affect their behavior. Therefore, many researchers argue that the teachers' attitudes have an effect on the students' performance and interests (e.g. see Ualesi and Ward, 2018). It could especially influence the teachers' preparedness for lessons (Omolara, and Adebukola, 2015). For that reason, it is important to take into account the opinion of teacher studies students concerning computer programming. This paper tries to make an acceptable classification model for distinguishing students who share the opinion that pupils should be acquainted with basic programming concepts during lower grades of primary education, from those students who do not share this opinion. The students' viewpoints concerning the use of programming in primary education was examined using the k-Nearest Neighbor (kNN) data mining method. It has effectively categorized 77.77% students who support the introduction of basic programming concepts at an early age, and 63.27% of those who do not. This model could be used by students and their educators as a means for improving communication and for encouraging discussions, but it could also be used as an indicator of whether additional student education is needed.

Keywords: kNN, programming, primary school, data mining, students

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Rudarenje stavova studenata o programiranju u osnovnom obrazovanju

Ivana Đurđević Babić i Dajana Sabolić

Fakultet za odgojne i obrazovne znanosti, Sveučilište u Osijeku, Hrvatska

Sažetak. U današnje vrijeme poželjno je steći kompetencije te znanja iz različitih područja računalnog programiranja. Dobro je poznato da nečija stajališta i stavovi utječu na njihovo ponašanje. Stoga, mnogi istraživači tvrde da stavovi nastavnika utječu na uspješnost učenika i njihov interes (npr. Vidi Ualesi i Ward, 2018.). Posebno, to bi moglo utjecati na pripremljenost nastavnika za nastavu (Omolara i Adebukola, 2015). Iz tog razloga, važno je uzeti u obzir razmišljanja studenata učiteljskog studija o računalnom programiranju. Osvrćući se na stavove studenata o upotrebi programiranja u primarnom obrazovanju, koristeći metodu knjubližih susjeda (kNN) kao metodu rudarenja podataka, ovaj rad pokušava napraviti prihvatljiv klasifikacijski model za razlikovanje studenata koji dijele mišljenje da bi učenici trebali biti upoznati s osnovnim konceptima programiranja u ranoj dobi primarnog obrazovanja, od onih studenata koji ne dijele to mišljenje. Rezultati su pokazali da je dobiveni kNN model ostvario željeni cilj i učinkovito kategorizirao 77.77% učenika koji podržavaju uvođenje osnovnih koncepata programiranja u ranoj dobi i 63.27% onih koji ne dijele to mišljenje. Ovaj model može poslužiti studentima i njihovim edukatorima kao sredstvo za poboljšanje komunikacije među njima i za poticanje rasprava, ali se može koristiti i kao pokazatelj je li potrebno dodatno obrazovanje studenata.

Ključne riječi: kNN, programiranje, osnovna škola, rudarenje podatcima, studenti

Simple mathematical model of Cyber-Physical System

Vjekoslav Galzina¹ and Roberto Lujic²

¹ Faculty of Education, University of Osijek, Croatia

² Department of Industrial Engineering, Mechanical Engineering Faculty, University of Osijek, Croatia

Abstract. The integration of networked computational (cyber) and hardware elements (physical) into systems is an important current and future challenge. Cyber-physical systems (CPS) are characterized by a strong union of hardware and software, where software implanted devices interact with the physical environment by means of sensors and actuators. Practical problems in governing such system is diversity of realms: on the one hand, the physical world is described by and based on continuous time dynamics, and on the other, hardware and software, both in computational and communication realm, are based on strict causality and discrete time dynamics. In this paper, a simple mathematical model is presented by using open-source hardware and software to be able to capture all realms of the proposed CPS.

Keywords: Cyber-Physical Systems, CPS model, continuous dynamics, discrete dynamics

Jednostavni matematički model kibernetičko-fizikalnog sustava

Vjekoslav Galzina¹ i Roberto Lujić²

¹ Fakultet za odgojne i obrazovne znanosti, Sveučilište u Osijeku, Hrvatska

² Zavod za industrijsko inženjerstvo, Strojarski fakultet u Slavonskom Brodu, Sveučilište u Osijeku, Hrvatska

Sažetak. Spajanje umreženih softverskih (eng. Cyber) i hardverskih resursa (eng. Physical) u integrirane sustave (eng. Systems) važan je trenutni problem i izazov budućnosti. CPS se odlikuje snažnom povezanošću hardvera i softvera, gdje softverski ugrađeni uređaji komuniciraju s fizičkim okruženjem pomoću senzora i aktuatora. Praktični problemi u upravljanju takvim sustavom proizlaze iz različitosti: s jedne strane imamo fizički svijet koji je zasnovan na kontinuiranoj vremenskoj dinamici, a s druge strane digitalni hardver i softver, koji se kako u računskoj tako i u komunikacijskoj sferi, temelji se na strogoj uzročnosti i diskretnoj vremenskoj dinamici. U ovom radu prikazan je jednostavan matematički model s korištenjem hardvera i softvera otvorenog koda kako bi se moglo obuhvatiti i integrirati sva područja predloženog CPS-a.

Ključne riječi: Kibernetički-fizikalni sustav, CPS model, kontinuirana dinamika, diskretna dinamika

What types of knowledge do mathematics textbooks promote?

Amanda Glavaš¹, Azra Staščik² and Ljerka Jukić Matić³

¹Mechanical Engineering School Osijek, Croatia

²Elementary school Dobriša Cesarić Osijek, Croatia

³Department of Mathematics, University of Osijek, Croatia

Abstract. This paper reports a study on the type of mathematical tasks and their overall representation in mathematics textbooks. The aim of the study was to investigate the ratio of tasks according to the main classification on procedural and conceptual tasks according to the type of knowledge one accessed to obtain a solution. The study provides further insight into the classification of each type of tasks by categories established by the researcher according to Rittle – Johnson and Schneider (2015) categorization. The study was conducted on two out of three mathematics textbooks approved by the Ministry of Science and Education of Republic of Croatia and covered two chapters from two different mathematics domains. The findings showed that procedural tasks were vastly present in investigated textbooks, unlike conceptual tasks. Based on these findings we argue for an increase of conceptual tasks in mathematics textbooks. Our argument stems from previous studies which advocate a balance of procedural and conceptual knowledge in teaching and learning mathematics. Further, we find the results disappointing since numerous studies show that textbooks play one of the main and almost central role in teachers' lesson preparation, as well as in the selection of tasks used during teaching phase and practicing phase of the lesson.

Keywords: procedural tasks, conceptual tasks, textbooks, teaching mathematics, textbooks' analysis

Koju vrstu znanja promoviraju matematički udžbenici?

Amanda Glavaš¹, Azra Staščik² i Ljerka Jukić Matić³

¹Strojarska i tehnička škola Osijek, Hrvatska

²Osnovna škola Dobriša Cesarić Osijek, Hrvatska

³Odjel za matematiku, Sveučilište u Osijeku, Hrvatska

Sažetak. Ovaj rad donosi rezultate istraživanja o vrsti matematičkih zadataka i njihovoј zastupljenosti u udžbenicima matematike. Cilj je istraživanja bio ustanoviti omjere zadataka prema jednoj od čestih podjela: na proceduralne i konceptualne zadatke. Rad također donosi uvid u dublju klasifikaciju svake od navedenih vrsta zadataka, koju su autori osmislili prema klasifikaciji danoj u Rittle – Johnson i Schneider (2015). Istraživanje je koncentrirano na matematičke udžbenike za srednje škole s četverogodišnjim programom. Istraživanje je provedeno na dva od tri prihvaćena i odobrena udžbenika, od strane Ministarstva znanosti i obrazovanje Republike Hrvatske, a pokrivena su dva različita matematička područja.. Rezultati pokazuju kako su proceduralni zadaci u udžbenicima uvelike zastupljeni, za razliku od konceptualnih zadataka. Vodeći se dobivenim rezultatima, zagovaramo povećanje konceptualnih zadataka u matematičkim udžbenicima. Naš zaključak proizlazi iz ranijih istraživanja koja pokazuju kako velika vrijednost leži u podjednakom poučavanju proceduralnog i konceptualnog znanja. Rezultate proizašle iz ovog istraživanja smatramo još nepovoljnijima jer mnoga istraživanja pokazuju kako su udžbenici jedan od glavnih elemenata koji utječu na pripremu nastave nastavnika, kao i na izbor zadataka koji će se koristiti za učenje i vježbanje.

Ključne riječi: proceduralni zadaci, konceptualni zadaci, udžbenici, poučavanje matematike, analiza udžbenika

Primary school teachers' (mis)understandings of equality and the equals sign

Željko Gregorović and Ana Katalenić

Faculty of Education, University of Osijek, Croatia

Abstract. The relation 'being equal to' and the equals sign are integral part of mathematics curriculum from the first years of education. However, research showed that students maintain misconceptions about equality and equals sign throughout their mathematics education. By examining mathematics textbooks for primary education (pupils of age 7–10) poorly designed tasks, mathematical inconsistencies, wrong labelling and other methodological omission can be found. In this paper we will present a textbook task with content contradictory to the notion of equality. We examined how in-service primary school teachers ($N=51$, between one and 34 years of experience) and university students, who are prospective primary school teachers ($N=340$), approached this task. Almost all participants in our survey interpreted the equals sign as 'perform calculation from left to right'. They mainly commented about the nature and structure of the task and rarely about the mathematical content of the task. In this paper we will present their judgments, experiences and observations related to the task. Observing that majority of prospective teachers showed misconception of equality we must consider addressing this issue through textbook revision, and both pre-service and in-service teacher education.

Keywords: equals sign, equality, mathematics textbook, primary school mathematics, mathematics teacher, teacher education

(Ne)razumijevanje jednakosti i znaka jednakosti kod učitelja razredne nastave

Željko Gregorović i Ana Katalenić

Fakultet za odgojne i obrazovne znanosti, Sveučilište u Osijeku, Hrvatska

Sažetak. Relacija "biti jednak" i znak jednakosti sastavni je dio kurikuluma predmeta matematike od početka obrazovanja. Međutim, istraživanja su pokazala da učenici zadržavaju zablude o jednakosti i znaku jednakosti tijekom njihovog matematičkog obrazovanja. Pregledavanjem udžbenika iz matematike za razrednu nastavu (učenici starosti 7-10 godina) mogu se pronaći loše osmišljeni zadaci, matematičke nedosljednosti, pogrešno označavanje i drugi metodički propusti. U ovom ćemo radu predstaviti zadatak iz udžbenika sa sadržajem koji je u suprotnosti s konceptom jednakosti. Ispitali smo kako su učitelji razredne nastave ($N = 51$, između jedne i 34 godine radnog iskustva) i studenti, koji su budući učitelji razredne nastave ($N = 340$), pristupili tom zadatku. Gotovo su svi sudionici u našem istraživanju protumačili znak jednakosti kao "izvrši izračun s lijeva na desno". Uglavnom su komentirali prirodu i strukturu zadatka, a rijetko matematički sadržaj. U ovom ćemo radu predstaviti njihove prosudbe, iskustva i zapažanja vezana uz zadatak. Primjećujući da je većina budućih učitelja pokazala pogrešnu predodžbu o jednakosti, moramo razmotriti rješavanje ovog pitanja kroz reviziju udžbenika, ali i kroz obrazovanje budućih učitelja te stručno i metodičko usavršavanje učitelja u praksi.

Ključne riječi: znak jednakosti, jednakost, udžbenik iz matematike, osnovnoškolska matematika, učitelj matematike, obrazovanje učitelja

Investigating adaptive reasoning and strategic competence in Croatian mathematics education: The example of quadratic function

Matea Gusić

Faculty of Teacher Education, University of Zagreb, Croatia

Abstract. Since the 1980s many mathematics educators have emphasized that being able to use computational procedures accurately and quickly, or reproduce large quantities of knowledge, isn't sufficient for students to be mathematically proficient. The development of students' reasoning and problem-solving skills, as well as the ability to connect and communicate mathematical ideas came into focus. One of the frameworks which considers a more comprehensive approach to mathematics learning is mathematical proficiency by Kilpatrick, Swafford and Findell. Mathematical proficiency proposes five equally important and mutually interdependent strands. This paper aims to investigate two of the strands: adaptive reasoning and strategic competence in Croatian mathematics education regarding quadratic functions. The objective is to gain insight into curriculum requirements and students' skills, explore indicators of adaptive reasoning and strategic competence and thus reach a deeper understanding of the situation. One of the most commonly used textbooks in high school mathematics in Croatia has been analysed with the purpose of determining the extent and the nature to which textbooks enable opportunities for the development of these strands regarding quadratic functions. A case study with three second-grade gymnasium students was conducted to gain insight into the adaptive reasoning and strategic competence skills activated while solving quadratic function tasks.

Keywords: mathematical proficiency, adaptive reasoning, strategic competence, quadratic function, textbook analysis, case study

Istraživanje prilagodljivog rasuđivanja i strateških kompetencija u hrvatskom matematičkom obrazovanju: Primjer kvadratne funkcije

Matea Gusić

Učiteljski fakultet, Sveučilište u Zagrebu, Zagreb, Hrvatska

Sažetak. Od osamdesetih godina 20. stoljeća ističe se da brzo i točno izvođenje procedura te reprodukcija velikih količina podataka, nisu dovoljni za razvoj matematički kompetentnog učenika. Naglasak se prebacuje na razvoj rasuđivanja, vještine rješavanja matematičkih problema te sposobnost povezivanja i komuniciranja matematičkih ideja. Primjer teoretskog okvira koji razmatra cjeloviti pristup nastavi matematike, naziva „matematičke kompetencije“ (Mathematical proficiency), ponudili su Kilpatrick, Swafford i Findell. Navedeni okvir predlaže pet jednakovražnih i međusobno povezanih dimenzija. Ovaj rad prikazuje istraživanje dviju dimenzija okvira, prilagodljivo rasuđivanje i strateške kompetencije u učenju i poučavanju sadržaja iz kvadratnih funkcija u Republici Hrvatskoj. Cilj istraživanja je dobivanje predodžbe u indikatore i razvijenost vještina prilagodljivog rasuđivanja i strateških kompetencija. Analizom jednog od najzastupljenijih matematičkih udžbenika za opće gimnazije dobiven je uvid u kurikularne zahtjeve u vidu opsega i vrste zadataka koji potencijalno promoviraju istraživane dimenzije. Studija slučaja, u kojoj su sudjelovale tri učenice drugog razreda opće gimnazijskog programa, omogućila je uvid u aktivaciju vještina prilagodljivog rasuđivanja i strateških kompetencija prilikom rješavanja zadatka iz kvadratne funkcije.

Ključne riječi: matematičke kompetencije, prilagodljivo rasuđivanje, strateške kompetencije, kvadratna funkcija, analiza udžbenika, studija slučaja

Poincaré's model of hyperbolic geometry

Anes Z. Hadžiomerović¹ and Amila Osmić²

¹ Second Gymnasium Mostar, Bosnia and Herzegovina

² Construction and Geodesy school of Tuzla, Bosnia and Herzegovina

Abstract. As a discipline, geometry has a long and rich history. Since as early as the ancient civilizations, e.g., for the sake of measuring the soil, it has developed as inductive science to take the important role in science today within the field of mathematics.

The great turning point and one of the most influential achievements of the nineteenth century is the discovery of non-Euclidean geometry. Carl Friedrich Gauss found the possibility of existence of non-Euclidean geometry. Hyperbolic geometry found acceptance with the work of mathematicians I. N. Lobachevsky and J. Bolay. There are many possible models for hyperbolic geometry. In this presentation, we are going to describe Poincaré's model of hyperbolic geometry.

Keywords: non-Euclidean geometry, hyperbolic geometry, Poincaré's disk model of hyperbolic geometry

Poincaréov model hiperboličke geometrije

Anes Z. Hadžiomerović¹ i Amila Osmić²

¹ Druga Gimnazija Mostar, Bosna i Hercegovina

² Građevinsko geodetska škola Tuzla, Bosna i Hercegovina

Sažetak. Geometrija ima dugu i bogatu povijest. Još od drevnih civilizacija, npr. radi mjerenja tla, razvila se kao znanost koja danas ima važnu ulogu u području matematike.

Velika prekretnica i jedno od najutjecajnijih dostignuća devetnaestog stoljeća je otkriće neeuklidske geometrije. Carl Friedrich Gauss je došao do spoznaje o mogućnosti postojanja neeuklidske geometrije. Hiperbolička geometrija bila je prihvaćena radovima matematičara I. N. Lobačevskog i J. Bolaya. Postoji mnogo mogućih modela za hiperboličku geometriju. U ovoj prezentaciji opisat ćemo Poincaréov model hiperboličke geometrije.

Ključne riječi: neeuklidska geometrija, hiperbolička geometrija, Poincaréov model hiperboličke geometrije

Generating question for Moodle base

Marija Jakuš and Lucija Žignić

Faculty of organization and informatics, University of Zagreb, Varaždin, Croatia

Abstract. Nowadays, the use of an LMS (learning management system) called Moodle is widespread. An LMS is in general a software application for administration, documentation, tracking, reporting and delivery of educational courses, training programs, or learning and development programs. At the Faculty of Organization and Informatics Varaždin, University of Zagreb, Moodle is also used to support the educational process.

On the course *Mathematics 1*, Moodle is used as a repository of presentations and documents, and for communication with students. There are also some activities on the course that are organized through Moodle. These are quizzes, whose results are used to generate student's final grade. Quizzes are individual for each student and generated randomly from a Moodle question bank. Question repetition frequency depends on the number of questions in the bank.

Questions can be added to the question bank by writing them directly into Moodle or they can be imported to the system from an external source. However, it takes time to write questions one by one. Luckily, some types of mathematics questions, whose answers can be calculated, can be generated in some programming language and imported into Moodle. In that way, the Moodle question bank can be easily expanded.

The aim of this article is to present how some types of questions can be generated by programming, and then included in Moodle.

Keywords: Moodle, mathematics question, programming, python, generating

Generiranje baze pitanja za Moodle

Marija Jakuš i Lucija Žignić

Fakultet organizacije i informatike, Sveučilište u Zagrebu, Varaždin, Hrvatska

Sažetak. Danas svjedočimo sve većoj upotrebi LMS (learning management system) sustava Moodle, sustava za upravljanje učenjem. LMS, općenito, je softverska aplikacija za podršku obrazovnim tečajevima, programima obuke, učenja i razvoja, koja omogućuje dostupnost, administriranje, dokumentiranje, praćenje i izvještavanje. Moodle se koristi i na Fakultetu organizacije i informatike Varaždin, sastavnici Sveučilišta u Zagrebu, kao podrška procesu učenja.

Na predmetu "Matematika 1", Moodle se koristi kao repozitorij nastavnih materijala te u komunikaciji sa studentima. U sustav su uključene i neke aktivnosti studenata na predmetu koje doprinose završnim ishodima. To su između ostalog testovi, čije se vrednovanje koristi u određivanju ocjene koju student dobiva iz predmeta na kraju semestra.

Testovi se generiraju za svakog studenta posebno tako da se iz baze pitanja nasumično odabere određeni broj pitanja. Frekvencija pojavljivanja određenog pitanja ovisi o broju pitanja koja se nalaze u bazi pitanja.

Kako bi oformili bazu, pitanja se mogu dodavati direktno pisanjem u sustav ili se mogu uvesti iz vanjskog izvora. Povećavanje baze, pišući jedno po jedno pitanje, zahtijeva mnogo vremena. Srećom, određena vrsta matematičkih pitanja, čiji se odgovori i rezultati mogu odrediti ili izračunati računalom, može se generirati u nekom programskom jeziku te dodati u Moodle. Tako se baza pitanja može lako povećati.

Cilj ovog izlaganja je prikazati kako se određena vrsta pitanja može programiranjem generirati i uključiti u sustav.

Ključne riječi: Moodle, pitanja iz matematike, programiranje, python, generiranje

The influence of teacher guides on classroom practice

Ljerka Jukić Matić¹ and Dubravka Glasnović Gracin²

¹ Department of Mathematics, University of Osijek, Croatia

² Faculty of Teacher Education, University of Zagreb, Croatia

Abstract. Textbooks and the accompanying teacher guides are recognized as important and influential resources in mathematics education. Teachers rely heavily on the textbook for planning and enacting their lessons: they prepare lessons according to the textbook structure, teach new content according to the textbook, and use the textbook as a source of practice exercises and homework. This study focuses on the role of teacher guides in teacher's classroom practice and their influence on the textbook use. The study also focused on the issue about how teachers change the utilization of textbooks in the classroom when the teacher guides undergo changes in their content and structure. Although the results showed relative stability of teaching practice, we detected an educative impact of teacher guides because their content influenced the use of active teaching methods in the classroom.

Keywords: classroom practice, longitudinal study, teacher, textbook, teacher guide

Utjecaj priručnika za učitelje matematike na nastavu matematike

Ljerka Jukić Matić¹ and Dubravka Glasnović Gracin²

¹ Odjel za Matematiku, Sveučilište u Osijeku, Hrvatska

² Učiteljski fakultet, Sveučilište u Zagrebu, Hrvatska

Sažetak. Udžbenici i pripadni priručnici za učitelje prepoznati su kao važni i utjecajni resursi u nastavi matematike. Učitelji se u velikoj mjeri oslanjaju na udžbenik prilikom planiranja, ali i samog izvođenje nastave: osmišljavaju nastavu prema strukturi udžbenika, podučavaju nove sadržaje prema udžbeniku i koriste udžbenik kao izvor za vježbanje i zadavanje domaće zadaće. Ova studija istražuje kakvu ulogu ima priručnik za nastavu matematike u nastavnoj praksi učitelja i kako priručnik utječe na samo korištenje udžbenika. Studija također istražuje kakav odnos imaju učitelji s udžbenikom ako se priručnik promijenio u svom sadržaju i strukturi. Iako su rezultati studije pokazali relativnu stabilnost nastavne prakse, otkrili smo da postoji edukativni utjecaj priručnika na učitelje; naime priručnik je potaknuo korištenje aktivnih nastavnih metoda u učionici.

Ključne riječi: nastavna praksa, longitudinalna studija, učitelj, udžbenik, priručnik za nastavu matematike

Student competence for solving logical tasks

Josipa Jurić¹, Irena Mišurac Zorica¹ and Maja Cindrić²

¹ Faculty of Philosophy, University of Split, Croatia

² Department of Teacher and Preschool Teacher Education, University of Zadar, Croatia

Abstract. Logical tasks do not require a great knowledge of math, but only require knowledge of basic concepts and logical considerations and conclusions. By inspecting math textbooks for primary and secondary schools we have noticed a considerably small number of them. Even when we did find such a task, it was conceptually or procedurally connected with the contents of the current learning. In order to find out whether the formal mathematical education is preparing students for solving of this kind of problems we have offered them five tasks. Surprisingly, we have discovered relatively small differences in the success of the primary and secondary schools, which indicates that the factual formal education does not contribute to the development of this skill. The analysis of some tasks has shown some surprising results such as, for example, that some of the early primary schools are more successful than older students.

Keywords: problem solving tasks, primary school, secondary school, formal mathematical education, logical conclusion

Pripremljenost učenika za rješavanje logičkih zadataka

Josipa Jurić¹, Irena Mišurac Zorica¹ i Maja Cindrić²

¹ Filozofski fakultet u Splitu, Sveučilište u Splitu, Hrvatska

² Odjel za izobrazbu učitelja i odgojitelja Sveučilišta u Zadru, Hrvatska

Sažetak. Logičkim zadatcima zovemo zadatke za čije rješavanje nisu potrebna velika matematička znanja već traže samo poznavanje osnovnih pojmoveva i logičkog promišljanja i zaključivanja. Uvidom u udžbenike matematike za osnovne i srednje škole uočili smo zanemarivo mali broj takvih zadataka. Čak i kad se pronađe neki logički zadatak on je konceptualno ili proceduralno vezan uz sadržaj trenutnog učenja. Kako bi zaključili priprema li formalno matematičko obrazovanje učenike za rješavanje ove vrste zadataka odabrali smo ih pet takvih životnog konteksta koje smo ponudili učenicima od drugog razreda osnovne škole do četvrtog razreda srednje škole. Iznenadile su nas relativno male razlike u uspješnosti rješavanja u dobnim skupinama učenika razredne i predmetne nastave u osnovnoj školi te srednjoškolaca što ukazuje na činjenicu da formalno obrazovanje nikako ne pridonosi dovoljno razvoju ove vještine. Analiza pojedinih zadataka pokazala je neke iznenađujuće rezultate kao na primjer da je u nekim zadatcima razredna nastava uspješnija od učenika zrelije dobi.

Ključne riječi: logički zadaci, razredna nastava, predmetna nastava, srednja škola, formalno matematičko obrazovanje, logičko zaključivanje

Pre-service teachers' prior knowledge related to measurement

Zdenka Kolar-Begović^{1,2}, Ružica Kolar-Šuper², Ivana Đurđević Babić² and Diana Moslavac Bičvić²

¹ Department of Mathematics, University of Osijek, Croatia

² Faculty of Education, University of Osijek, Croatia

Abstract. It is of great importance to use mathematical knowledge to solve real problems. Certainly one of the fields of mathematics that plays a major role here is geometry. This paper examines prior knowledge of pre-service teachers in relation to the concept of measurement. Basic knowledge in this field is necessary for a student educated to teach mathematics in the first four grades of primary school as this content is part of the mathematics curriculum for the given period. The participants of the study were first-year pre-service teachers, i.e., a total of 103. The paper focuses on analyzing students' responses to questions referring to measurement. Research results indicate that there are deficiencies in the knowledge of this field, which points to the need for additional increased attention to this concept in the courses offered to students in their studies, thus contributing to increasing the level of knowledge in this field.

Keywords: geometry, measurement, measurement units, pre-service teacher, mathematical concepts

Predznanje studenata učiteljskog studija vezano uz koncept mjerenja

Zdenka Kolar-Begović^{1,2}, Ružica Kolar-Šuper², Ivana Đurđević Babić² i
Diana Moslavac Bičvić²

¹ Odjel za matematiku, Sveučilište u Osijeku, Hrvatska

² Fakultet za odgojne i obrazovne znanosti, Sveučilište u Osijeku, Hrvatska

Sažetak. Velika je važnost korištenja matematičkog znanja u rješavanju realnih problema. Svakako jedno od područja matematike koje u tome igra veliku ulogu je područje geometrije. U ovom članku istražuje se predznanje studenata učiteljskog studija vezano uz koncept mjerenja. Osnovna znanja iz ovog područja nužna su za kadar koji se obrazuje za poučavanje matematike u prva četiri razreda osnovne škole s obzirom na zastupljenost ovih sadržaja u programu matematike za navedeno razdoblje. Sudionici istraživanja su studenti prve godine učiteljskog studija, njih 103. Rad je usmjeren na analizu odgovora studenata na pitanja iz domene mjerenja. Rezultati istraživanja upućuju na to da postoje manjkavosti u predznanju iz ovog područja, što upućuje na potrebu dodatnog posvećivanja pažnje ovom konceptu na kolegijima koji se studentima nude na studiju, čime bi se doprinijelo povećanju razine znanja iz ovog područja.

Ključne riječi: geometrija, mjerenje, mjerne jedinice, studenti učiteljskog studija, matematičke domene

The use of mental geometry in the development of the geometric concept of rotation

Nikolina Kovačević

Faculty of Mining, Geology and Petroleum Engineering, Department of Mathematics, Informatics and Descriptive Geometry, University of Zagreb, Croatia

Abstract. One of the most important roles of education is to improve the ability to solve real-life problems. Since a rigid transformation of rotation is one such example in a number of geological situations where structural lines and planes have been rotated from some initial spatial orientation, the notion of rotation takes an important place in the education of geologists. The paper presents the author's teaching experience in developing the concept of rotation as one of the basic geometry concepts within two undergraduate geometry courses in the field of technical sciences at the University of Zagreb.

Combining the principles of classical geometrical representation methods supported by new media, both courses extensively tend to use mental geometry to help students solve many geometry problems that arise from field observations. The aim of the paper is twofold. On the one hand, it presents typical problems that teachers encounter in the teaching of geometry, along with some concrete examples that suggest how problems can be successfully solved by following the latest methodological achievements in modern geometry education supported by mental geometry. On the other hand, the author intends to draw attention to difficulties in the teaching of some simple geometry concepts that are possibly caused by lack of operational knowledge of geometry of students in the field of technical sciences in Croatia.

Keywords: teaching geometry, mental geometry, spatial geometry, rotation, conceptual knowledge

Korištenje mentalne geometrije u razvijanju geometrijskog koncepta rotacije

Nikolina Kovačević

Rudarsko-geološko-naftni fakultet, Zavod za matematiku, informatiku i nacrtnu geometriju,
Sveučilište u Zagrebu, Hrvatska

Sažetak. Jedna od najvažnijih uloga obrazovanja je poboljšati sposobnosti rješavanja stvarnih problema. Kako je kruta transformacija rotacije jedan takav primjer u brojnim geološkim situacijama gdje se primarne strukture pravaca i ravnina rotiraju iz nekog inicijalnog prostornog položaja, pojам rotacije zauzima istaknuto mjesto u obrazovanju studenata preddiplomskog studija geološkog inženjerstva. Rad predstavlja nastavničko iskustvo u razvijanju koncepta rotacije, jednog od temeljnih geometrijskih koncepata, u okviru dva geometrijska kolegija na preddiplomskoj sveučilišnoj razini studija iz područja tehničkih znanosti na Sveučilištu u Zagrebu.

Kombinirajući principe klasične geometrijske metode reprezentacije potpomognute upotrebom novih medija, oba kolegija u velikoj mjeri koriste mentalnu geometriju kako bi pomogla studentima u rješavanju geometrijskih problema koji proizlaze iz promatranja na terenu. Namjera ovog rada je dvojaka. S jedne strane, prikazani su tipični problemi s kojima se susreću nastavnici pri poučavanju geometrijskih sadržaja, kao i neki konkretni primjeri koji upućuju na to kako se problemi mogu uspješno riješiti sljedeći najnovija metodološka dostignuća u suvremenoj metodici nastave matematike kroz primjenu mentalne geometrije. S druge strane, autor namjerava skrenuti pozornost na poteškoće u podučavanju nekih jednostavnih geometrijskih koncepata koje su vjerojatno uzrokovane nedovoljnom razinom operativnog znanja studenata iz područja tehničkih znanosti u Hrvatskoj.

Ključne riječi: poučavanje geometrije, mentalna geometrija, geometrija prostora, rotacija, konceptualno znanje

How do novices and experts approach an open problem?

Zoltán Kovács¹ and Eszter Kónya²

¹ Institute for Mathematics and Computer Science, University of Nyíregyháza, Hungary

² Institute for Mathematics, University of Debrecen, Hungary

Abstract. There are well-known differences between problem solving competencies of novices and experts. It is also known from the literature that students, regardless of age, typically give the expected answer, ignoring the openness of the problem. In our study, we use a problem that is open at the starting point nevertheless it has closed solution. We analyze how experts and novices manage the openness of an elementary problem based on a survey prepared for 7th and 8th graders.

Keywords: open problem, realistic reaction, novice and expert problem solver

Hogyan közelítenek meg egy nyitott problémát a kezdők és a szakértők?

Kovács Zoltán¹ és Kónya Eszter²

¹ Nyíregyházi Egyetem, Matematika és Informatika Intézet, Nyíregyháza, Hungary

² Debreceni Egyetem, Matematika Intézet, Debrecen, Hungary

Absztrakt. A kezdők és a szakértők problémamegoldó kompetenciái között jól ismertek a különbségek. Az irodalomból is ismert, hogy a tanulók az életkortól függetlenül jellemzően az elvárt választ adják, figyelmen kívül hagyva a probléma nyitottságát. Vizsgálatunkban egy olyan problémát tüztünk ki, amely a kiindulási ponton nyitott, de zárt megoldása van. Elemezzük, hogy 7. és 8. osztályos, kezdőnek, illetve szakértőnek számító tanulók hogyan reagálnak a probléma nyitottságára.

Kulcsszavak: nyitott problémák, realisztikus reakció, kezdő és szakértő problémamegoldó

Mathematical abilities of pre-school children

Ljiljanka Kvesić and Slavica Brkić

Faculty of Natural Sciences, Mathematics and Education, University of Mostar, Bosnia and Herzegovina

Abstract. Regardless of the fact that people are born with an innate sense of numbers, mathematical thinking requires a certain intellectual effort for which many children are not ready. While children investigate and discover new issues in everyday life, they meet the world of mathematics although they are not aware of it. Although mathematics is becoming more and more important today in the of advanced technology, it is very important that children are introduced to the spells of mathematics before they start attending school, and to continue to learn mathematics with that knowledge throughout their education. Just like reading, mathematics is a subject necessary for adequate functioning in society. Moreover, mathematics is a subject that develops logical thinking and perception, thus mathematical teaching of children ought to be placed on a more accessible level than it is currently. Parents and educators have access to various games and activities that involve children into mathematical thinking and creative resolving, which develops their self-confidence.

Through research conducted with both pre-school and primary school children using various mathematical examples appropriate for their age, it has been found out that the children of younger age have a sense of numbers and quantity, whereas primary school children base their conclusion upon the “expected”.

Keywords: child, sense of numbers and quantity, mathematics, mathematics through playing

Matematičke sposobnosti djece predškolske dobi

Ljiljanka Kvesić i Slavica Brkić

Fakultet prirodoslovno-matematičkih i odgojnih znanosti, Sveučilište u Mostaru, Bosna i Hercegovina

Sažetak. Bez obzira na to što se rađamo s urođenim osjećajem za broj, matematičko razmišljanje zahtjeva određeni misaoni napor za koji mnoga djeca nisu spremna. Dok djeca istražuju i otkrivaju nove stvari u svakodnevnom životu, oni se susreću sa svijetom matematike iako toga nisu svjesni. Budući da matematika postaje sve važnija u današnjem vremenu tehnologije, veoma je važno da djeca, upoznaju čari matematike dok još nisu krenula u školu i da s tim spoznajama nastave učiti matematiku tijekom školskog obrazovanja. Kao i čitanje, matematičko znanje je neophodno za adekvatno funkcioniranje u društvu. Čak i više od toga, matematika je predmet koji razvija logičko razmišljanje i percepciju, pa bi matematičko poučavanje djece trebalo biti na pristupačnijoj razini nego što trenutno jeste. Roditeljima i odgajateljima dostupne su razne igre i aktivnosti koje uključuju djecu u matematičko razmišljanje i kreativno rješavanje čime se razvija njihovo samopouzdanje.

U istraživanju koje je provedeno s djecom predškolske dobi i djecom primarnog obrazovanja, na različitim matematičkim primjerima prikladnim za njihovu dob, došlo se do spoznaje da djeca mlađeg uzrasta imaju osjećaj za broj i količinu, dok djeca primarnog obrazovanja svoj zaključak temelje na „očekivanju“.

Ključne riječi: dijete, osjećaj za broj i količinu, matematika, matematika kroz igru

Computer-based assessments in mathematics at the higher education level

Josipa Matotek

Faculty of Civil Engineering and Architecture, University of Osijek, Croatia

Abstract. An important part of teaching and learning process is evaluation of students' knowledge. Often, evaluation of mathematical knowledge at university level is carried out in two phases. The first phase is evaluation of task-solving ability and the second phase is the evaluation of mathematical theoretical knowledge. Due to an increase in the use of Information and Communication Technologies (ICT) in teaching and learning processes, there are more and more examples of computer-based assessments.

The paper presents the author's years of experience in using computer-based assessments since the author has been used them (in second phase of evaluation), in Learning Management System named Moodle since 2011, within the course Mathematics for Engineers 1 of the undergraduate professional study program at the Faculty of Civil Engineering and Architecture, Osijek. The paper points out the differences between paper-and-pencil and computer-based assessments, referring to different approach to creating questions and ways of implementing the exam itself. Additionally, it discusses the results of computer-based exams, and gather students' overall impression of this type of assessment.

Finally, the aim of this paper is to highlight the advantages and disadvantages of using computer-based assessments in mathematics at the higher education level.

Keywords: evaluation of knowledge, mathematics, computer-based assessment, paper and pencil exam, higher education level

Provjera znanja iz matematike na računalu na razini visokog obrazovanja

Josipa Matotek

Građevinski i arhitektonski fakultet, Sveučilište u Osijeku, Hrvatska

Sažetak. Bitan dio nastavnog procesa je i evaluacija znanja studenata. U matematici na visokoškolskoj razini često se to vrednovanje provodi u dvije faze. Prva faza je procjena znanja rješavanja zadataka, a druga faza je procjena teroretskog znanja. Uslijed sve većeg uvođenja informacijsko komunikacijskih tehnologija (IKT) u školski sustav na različite načine, sve je više primjera evaluacije znanja koristeći računala u svrhu testiranja znanja.

U ovom radu je opisano višegodišnje autorovo iskustvo u korištenju računala u svrhu ispitivanja znanja. Autor je u sklopu svoje nastave iz kolegija Matematika za inženjere 1 na stručnom studiju na Građevinskom i arhitektonskom fakultetu koristio testove (u ispitivanju teoretskog znanja) kreirane u sustavu za upravljanje učenjem, Moodle-u, od 2011. godine. U radu će biti opisane razlike između klasičnih testova (pisanih na papiru) i testova koji se provode na računalima, pri tome misleći na bitno različite načine kreiranja pitanja i načine provedbe samog ispita. Nadalje, bit će komentirani rezultati ispita dobiveni na ovakav način te prikazani utisci studenata o ovakovom načinu provođenja ispita.

Cilj rada je istaknuti prednosti i nedostatke korištenja računala u testovima iz matematike na visokoškolskoj razini.

Ključne riječi: evaluacija znanja, matematika, testovi na računalu, klasičan ispit, razina visokog obrazovanja

Redesigning a contextual textbook task with an exponential-type function using *a posteriori* analysis of the prospective mathematics teachers' work

Željka Milin Šipuš¹, Aleksandra Čižmešija¹ and Ana Katalenić²

¹Faculty of Science, University of Zagreb, Croatia

²Faculty of Education, University of Osijek, Croatia

Abstract. An exponential function is one of the elementary functions in mathematical modelling and a common part of the upper secondary mathematics curriculum. A horizontal asymptote is a prominent feature of an exponential function graph. We have performed a comprehensive study within the Anthropological theory of the didactics on asymptotes and asymptotic behaviour in the context of upper secondary mathematics education in Croatia. Therein, we administered three questionnaires with prospective mathematics teachers. One of the tasks from the questionnaires was a priori designed to explore the role of an asymptote in graphing and describing an exponential function. In this paper, we present results of an *a posteriori* analysis of students' work on the modified contextual textbook task with an exponential-type function. We discuss the affordances of the textbook versus redesigned task and provide suggestions for task selection and design to promote coherency and discourse in students' available mathematical knowledge.

Keywords: Anthropological theory of the didactics, *a posteriori* analysis, contextual task, exponential function, function graph, task design

Redizajniranje kontekstualnog udžbeničkog zadatka s funkcijom eksponencijalnog tipa pomoću *a posteriori* analize radova studenata nastavničkog studija matematike

Željka Milin Šipuš¹, Aleksandra Čižmešija¹ i Ana Katalenić²

¹ Prirodoslovno-matematički fakultet, Sveučilište u Zagrebu, Hrvatska

² Fakultet za odgojne i obrazovne znanosti, Sveučilište u Osijeku, Hrvatska

Sažetak. Eksponencijalna funkcija je jedna od temeljnih funkcija u matematičkom modeliranju i uglavnom je dijelom kurikuluma srednjoškolskog matematičkog obrazovanja. Horizontalna asimptota je istaknuto svojstvo grafa eksponencijalne funkcije. Proveli smo opsežnu studiju unutar Antropološke teorije didaktike o asimptoti i asymptotskom ponašanju u kontekstu srednjoškolskog matematičkog obrazovanja u Hrvatskoj. Dio te studije čine upitnici s budućim učiteljima matematike. Jedno od pitanja iz upitnika je *a priori* dizajnirano za ispitivanje uloge asimptote u crtaju i opisivanju eksponencijalne funkcije. U ovom radu, predstavljamo rezultate *a posteriori* analize studentskih uradaka na izmijenjenom udžbeničkom zadatku s funkcijom eksponencijalnog tipa. Raspravljamo o svojstvima udžbeničkog i redizajniranog zadatka te donosimo prijedloge za izbor i dizajn zadatka koji doprinose razvijanju koherentnosti i diskursa u matematičkom znanju studenata.

Ključne riječi: Antropološka teorija didaktike, *a posteriori* analiza, dizajn zadatka, eksponencijalna funkcija, graf funkcije, kontekstualni zadatak

The role of online applications as a tool of support in mathematics education

Ana Mirković Moguš

Faculty of Education, University of Osijek, Croatia

Abstract. Mathematics plays an important role in the educational and developmental aspects of each country. Many reports show that students have difficulty learning mathematics. There is some evidence that suggests technology can promote learning mathematics and improve student performance in this area. The aim of this paper is to review the research on the use of online applications for teaching and learning mathematics. Along with investigating the use of applications for mathematics, their influence on students' mathematical learning experience is researched as well. The results of the review implicate a basis for choosing a framework for evaluation and informing teaching decisions about the use of applications to enhance students' conceptual understanding. The framework can be used as a support for teachers' professional development. Results also show that online applications offer benefits in learning and teaching mathematics, but it also seems that a set of technological and pedagogical elements is necessary for the enhancement of mathematics active inclusion, learning and mathematical thinking.

Keywords: technology, support, teacher, mathematics, education

Uloga online aplikacija kao alata podrške u matematičkom obrazovanju

Ana Mirković Moguš

Fakultet za odgojne i obrazovne znanosti, Sveučilište u Osijeku, Hrvatska

Sažetak. Matematika ima važnu ulogu u obrazovnim i razvojnim aspektima svake države. Mnogi izvještaji prikazuju da učenici imaju poteškoća u učenju matematike. Postoje neki dokazi koji upućuju na tehnologiju kao pomoć u promicanju učenja matematike i poboljšanju uspješnosti učenika u matematici. Cilj je ovog rada dati pregled istraživanja o korištenju online aplikacija za poučavanje i učenje matematike. Osim toga, ispitan je i utjecaj online aplikacija na iskustvo učenja matematike od strane učenika. Rezultati pregleda daju osnovu za odabir okvira za evaluaciju i informiranje učiteljskih odluka o korištenju aplikacija u svrhu poboljšanja učeničkog konceptualnog razumijevanja. Okvir se može koristiti i kao podrška stručnom razvoju učitelja. Rezultati također upućuju da online aplikacije nude prednosti u učenju i poučavanju matematike, ali je također potreban i skup tehnoloških i pedagoških elemenata nužan za poboljšanje aktivnog uključivanja u matematici, matematičkog učenja i razmišljanja.

Ključne riječi: tehnologija, podrška, učitelj, matematika, obrazovanje

From a nice tiling to theory and applications

Emil Molnár, István Prok and Jenő Szirmai

Department of Geometry, Institute of Mathematics, Budapest University of Technology and Economics, Hungary

Dedicated to the Memory of Professor Stanko Bilinski (1909-1998).

Abstract. Starting with the fundamental tiling for the Euclidean plane group 16.
p6 with pentagons, we see that any tile – after “logical” side pairing (as gluing) – will
be a surface (now a topological sphere, realized by a doubly covered triangle, of
angles $90^\circ = \pi/2$, $60^\circ = \pi/3$, $30^\circ = \pi/6$) with singular points (as above by periods 2,
3, 6, respectively) as rotational centres with angles $180^\circ = \pi$, $120^\circ = 2\pi/3$, $60^\circ = \pi/3$
(imagine with colours: green, blue, red), respectively.

Consider this for the 17 plane groups of Euclidean plane E^2 , then for the
analogous groups of sphere S^2 (where the angle sum of any triangle bigger than 180°
= π ; we have infinitely many groups, but finitely many types), moreover for the
analogous groups of the Bolyai-Lobachevsky hyperbolic plane H^2 (where the angle
sum of any triangle smaller than π ; infinitely many groups, but we shall have an
overview by the so called Macbeath signature).

And so on, going to the classical theory, relatively completed for dimensions 2
in our joint work (Lučić et al., 2018) on so-called Poincaré–Delone (Delaunay)
problem, and its nice applications in science and arts. The very hard analogous 3-
dimensional topic will also be mentioned, mainly with examples and figures only.

Our aim is also to give an Honour to the Memory of Professor Stanko Bilinski,
a scientific Father of former Yugoslavian and Hungarian geometers, who was a
“master of Brezelfläche” (double torus) and its hyperbolic tilings, see e.g. in
(Bilinski, 1985).

Keywords: crystallographic groups in E^2 , S^2 , H^2 , Poincaré–Delaunay problem
of “planigons”

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Egy szép kövezéstől az elméletig, az alkalmazásokig

MOLNÁR Emil, PROK István és SZIRMAI Jenő

Budapesti Műszaki és Gazdaságtudományi dományi Egyetem, Matematika Intézet, Geometria Tanszék, Hungary

Stanko Bilinski Professzor Úr (1909-1998) Emlékére

Kivonat. Az 1. ábrán (képzeljük el a színeket) az \mathbf{E}^2 euklideszi sík ötszög-alaptartományokkal történő szabályos kövezését láthatjuk. Vegyük észre a hatod-rendű α forgatásokat piros-színű középpontokkal, $2\pi/6 = 60^\circ$ szögekkel, és hasonlóan a kék-színű 3-centrumokat (β), zöld-színű 2-centrumokat (γ), a megfelelő 120° , illetve 180° forgásszögekkel. A 6-centrumok mutatják a fellépő eltolások 6-forgású rácsát is. Bármely két ötszöghöz létezik egyetlenegy olyan egybevágósági transzformáció, mely az első ötszöget a másodikba viszi úgy, hogy az egész kövezés önmagára képződik. A transzformációk összessége, melyek mindegyike *egy tetszőlegesen rögzített (sárga) (1)-tartományt egy tetszőleges g-tartományba képez, alkotja a $G = \mathbf{p6}$ ($g \in G$) jelű csoportot a kompozíció* (egymásutáni végrehajtás) műveletével. Így a fenti G csoportról is geometriai képet alkottunk: Egy út, mely az (1) tartománytól a szomszédokon át a *g tartományig vezet*, a g csoportelemet az α, β, γ generátorokkal (és azok $\alpha^{-1}, \beta^{-1}, \gamma^{-1} = \gamma$ inverzeivel) fejezi ki (nem könnyen!). Így képezhetjük a *G csoport-gráfot háromfélé (színes) irányított éssel, a három generátornak megfelelően*.

Képzeljék el, hogy a csoportelmélet egészét ilyen geometriai nyelven is megfogalmazhatjuk, és ez a szemléltetés sok előnnyel jár, vonzó geometriai problémákhoz vezet. Például, a fenti $\mathbf{p6} = 236$ csoportnak 4-féle (*topológikus*) típusú alaptartománya van (keressük meg a sokszög-típusokat!). A 30-, 60-, 90-fokos iskolai vonalzó sokat segíthet. Képzeljük el az analóg 234, 235 csoportok alaptartományait, ott is 4 típus van, de már az \mathbf{S}^2 jelű gömbfelületen; vagy a 237, 238, ... csoportok végtelen sok esetét, ott is 4 típus van, de már a Bolyai-Lobacsevszkij-féle \mathbf{H}^2 hiperbolikus síkon. A jól ismert ($45^\circ, 45^\circ, 90^\circ$) iskolai vonalzó segíthet a $G = 244 = \mathbf{p4}$ csoport 3-féle alaptartományának megtalálásában.

Az E^2 euklideszi síkon 17 analóg síkcsoport van (a középkori iszlám művészet zseniális intuitív felfedezése nyomán, pl. a granadai Alhambra (Vörös vár) kolostortemplom díszítései, Spanyolország). Pólya György, híres magyar matematikus (és matematika-tanár) kedvenc témái is voltak ezek a csoportok. Ezek alaptartományainak mind a 46 topológikus típusát (és a további 47 un. laptranzitív kövezést) csak 1959-ben publikálta B. N. Delone (Delaunay) orosz matematikus. H. Poincaré már 1882-ben megkísérelte a H^2 hiperbolikus sík analóg csoportjainak leírását, a C komplex számsíkon modellezve H^2 -t, és alkalmas alaptartományok keresésével (később kiderült, hogy kísérlete “eleve reménytelen” volt, de módszerét a térré is kiterjesztette).

Az a *Poincaré-Delaunay probléma*, hogy az S^2 , E^2 , H^2 síkokon egységesen megkeressük az analóg csoportok (kompakt, azaz korlátos és zárt) alaptartományait, hosszú időre nyitott kérdés maradt.

Stanko Bilinski professzor úr, a Zágrábi Egyetem egykori geometria tanára, az akkor jugoszláv és a magyar geométerek tudományos édesapja ugyancsak sokat foglalkozott a hiperbolikus síkcsoportokkal és azok realizációival. Kedvence volt a duplatórusz (perec-felület vagy Brezelfläche, szép német nyelvű előadásaiban), lásd [1] csak példaként.

Az “utolsó lépés”, előadásunk fő témája, lehet talán az a számítógépes program, a *program COMCLASS* és az az összefoglaló cikk [2], melyet nem-rég publikáltunk.

Az előadásban röviden, a konferencia-kötetben kicsit hosszabban emlíjtük majd a szerzők térbeli eredményeit. A térbeli megfelelő problémák általában nyitottak, néha “reménytelenek” túnnek.

Kulcsszavak: szabályos kövezések, síkcsoportok a gömbön, az euklideszi és a hiperbolikus síkon

Teachers' opinions on geometric contents in the curriculum for the lower grades of primary school

Sanela Nesimović¹ and Karmelita Pjanić²

¹ Faculty of Educational Sciences, University of Sarajevo, Bosnia and Herzegovina

² Faculty of Pedagogy, University of Bihać, Bosnia and Herzegovina

Abstract. In order to obtain a higher quality of education, each one of its segments should be maximally arranged. Teachers are an essential part of quality education in mathematics. Their role is irreplaceable: the way they teach and think about their profession directly influences the results of their work, i.e. the learning outcomes of their pupils. The manner and the quality of their work depend on their opinion (or attitude) towards other segments of the educational system in mathematics.

The aim of this paper is to investigate teachers' opinions on geometric contents in the curriculum for the first five grades of primary school and the quality of mathematics textbooks for the first five grades, along with their reflections on the teaching methods used in teaching the geometric contents. For this purpose, the survey was conducted among 108 teachers from 11 primary schools in Sarajevo Canton.

By analyzing the results of the survey, we can notice that the teachers of different educational levels and years of work experience had the same or similar opinions to all the questions dealing with the representation of geometric contents, the quality of textbooks for a particular grade as well as the selection and application of teaching methods in teaching geometry.

Keywords: geometry, primary education, teachers' opinions, textbook, teaching methods

Mišljenja učitelja o geometrijskim sadržajima zastupljenim u nižim razredima

Sanela Nesimović¹ i Karmelita Pjanić²

¹ Pedagoški fakultet, Univerzitet u Sarajevu, Bosna i Hercegovina

² Pedagoški fakultet, Univerzitet u Bihaću, Bosna and Hercegovina

Sažetak. Da bismo imali što kvalitetnije obrazovanje, svaki njegov segment bi trebao biti maksimalno uređen. Bitnu kariku kvalitetnog matematičkog obrazovanja čine učitelji. Njihova uloga je nezamjenjiva: način na koji rade i kako razmišljaju o svom pozivu direktno utiče na rezultate njihovog rada, odnosno na ishode učenja kod učenika. Način i kvalitet njihovog rada zavisi od njihovog mišljenja (ili stava) prema preostalim segmentima matematičkog obrazovanja.

Cilj ovog rada je da istražimo mišljenja učitelja o geometrijskim sadržajima u kurikulumu za prvih pet razreda osnovne škole, o kvaliteti udžbenika matematike za prvih pet razreda, te mišljenja o nastavnim metodama koje se koriste pri poučavanju geometrijskih sadržaja. U tu svrhu anketirano je 108 učitelja iz 11 osnovnih škola Kantona Sarajevo.

Analizom rezultata anketiranja možemo primijetiti da su učitelji različitog obrazovnog nivoa i godina staža imali ista ili slična mišljenja za sva postavljena pitanja koja se odnose na zastupljenost geometrijskih sadržaja, kvaliteti udžbenika za određeni razred kao i odabiru i primjeni nastavnih metoda u procesu poučavanja geometrije.

Ključne riječi: geometrija, razredna nastava, mišljenje učitelja, udžbenik, nastavne metode

Geometric thinking of primary school pupils

Sanela Nesimović¹ and Karmelita Pjanic²

¹ Faculty of Educational Sciences, University of Sarajevo, Bosnia and Herzegovina

² Faculty of Pedagogy, University of Bihać, Bosnia and Herzegovina

Abstract. According to Pierre van Hiele's theory of the levels of geometric thinking, there are five levels of geometric thinking: visualization, analysis, informal deduction, deduction and rigor. Those levels are not conditioned by one's age, but their order is unchangeable. On the other hand, the teaching of mathematics is characterized by the fact that pupil deal with various mathematical concepts adopted at different levels at the same time. Some of the concepts have been developed completely while the development of other concepts has remained at the level of mere perception.

In this paper, we propose an age-appropriate framework for reaching each of the van Hiele's levels, based on van Hiele's theory of geometric thinking and taking into account the document 'The Common Core Curricula for Mathematics Based on Learning Outcomes in Bosnia and Herzegovina'. The aim of this paper is to determine whether primary school pupils can identify the elements of different levels of geometric thinking according to van Hiele. For this purpose, a survey was carried out among 1889 primary school pupils (1st - 5th grade) from ten primary schools in Sarajevo Canton. For the purpose of testing, the diagnostic tests were created containing the tasks which detect the elements of the following levels: visualization, analysis, informal deduction.

The research results show that the levels of geometric thinking of primary school pupils generally fit into the suggested framework. In addition, it was noticed that pupils who achieve the level of visualization and analysis succeed in detecting causal relationships regarding the specific geometric content. The obtained results indicate that the transition from one level of geometric thinking to the next one does not happen suddenly.

Keywords: geometric thinking, primary school pupils, visualization, analysis, informal deduction

Geometrijsko mišljenje učenika razredne nastave

Sanela Nesimović¹ i Karmelita Pjanić²

¹ Pedagoški fakultet, Univerzitet u Sarajevu, Bosna i Hercegovina

² Pedagoški fakultet, Univerzitet u Bihaću, Bosna and Hercegovina

Sažetak. Prema van Hieleovoj teoriji o nivoima geometrijskog mišljenja, postoji pet nivoa geometrijskog mišljenja: vizualizacija, analiza, neformalna dedukcija, dedukcija i strogost. Ti nivoi nisu uslovjeni godinama starosti, ali njihov poredak je nepromjenljiv. S druge strane, nastavu matematike karakteriše to da učenici u isto vrijeme barataju različitim matematičkim pojmovima koji su usvojeni na različitim nivoma. Neki od pojmoveva su razvijeni u potpunosti dok je razvoj drugih pojmoveva ostao na nivou same percepcije.

U ovome radu predlažemo jedan uzrasno prikladni okvir za dostizanje svakog od van Hieleovih nivoa, zasnovano na van Hieleovoj teoriji geometrijskog mišljenja i uzimajući u obzir dokument 'Zajednička jezgra nastavnih planova i programa za matematiku bazirano na ishodima učenja za Bosnu i Hercegovinu'.

Cilj ovog rada je ustanoviti da li se kod učenika razredne nastave mogu identifikovati elementi različitih nivoa geometrijskog mišljenja prema van Hieusu. U tu svrhu provedeno je istraživanje među 1889 učenika osnovne škole (od 1. do 5. razreda) iz deset osnovnih škola Kantona Sarajevo. U cilju ispitivanja kreirani su dijagnostički testovi, koji sadržavaju zadatke koji otkrivaju elemente sljedećih nivoa: vizualizacija, analiza, neformalna dedukcija.

Rezultati istraživanja pokazuju da nivoi geometrijskog mišljenja kod učenika razredne nastave generalno odgovaraju predloženom okviru. Osim toga, uočeno je da učenici koji dosegnu nivo vizualizacije i analize uspijevaju da uviđaju kauzalne odnose u pogledu specifičnog geometrijskog sadržaja. Dobiveni rezultati ukazuju na to da se prelazak s jednog nivoa geometrijskog mišljenja na sljedeći ne odvija u naglim skokovima već kontinuirano.

Ključne riječi: geometrijsko mišljenje, razredna nastava matematike, vizualizacija, analiza, neformalna dedukcija

The principle of mathematical induction and Peano's axioms, their definition and application through the prism of the methods of mathematics and mathematical competences of the mathematics teachers

Ahmed Palić¹, Maid Omerović¹ and Edisa Korda²

¹ Faculty of Education, University of Travnik, Bosnia and Herzegovina

² Elementary school “Turbe“ in Travnik, Bosnia and Herzegovina

Abstract. In our talk, we will explore concrete and necessary theoretical and practical basics related to the Principle of Mathematical Induction and Peano's Axioms, their methodology and the level of student knowledge related with them. First, we will define mathematical induction and present some consequences. Next, we will provide several examples, to show the easiest possible way of understanding of mathematical induction as well as Peano's Axioms. Further, we will present a methodology of the researched problem. Then, we will present the analysis and the results of the research, through the prism of the mathematical methods and related with mathematical competence of the mathematics teachers. Finally, we will solve the same problems. The participants in the research were students coming from second, third and fourth year of the Faculty of Education, University of Travnik. Their distribution was following: the sample consist of 15.4% of second year students, 23.1% of third year students, and 61.5% of fourth year students. The students belong to two different cohort; 84.6% of students coming from the Department of Mathematics and Informatics and 15.4% from the Department of Classroom Teaching. In the research, we will use the analytical-descriptive method, the method of theoretical analysis, the causal method, and the survey method. The results of the research will be presented in a graphical and tabular form with explanation and discussion. Our talk will give an overview of the current state of the theoretical and practical basis of the Principle of Mathematical Induction and Pean's Axioms, as well as the level of knowledge of students in this field.

Keywords: principle, induction, mathematics, axiom, methods, competences

Princip matematičke indukcije i Peanovi aksiomi, njihova definicija i primjena kroz prizmu metoda matematike i matematičkih kompetencija nastavnika matematike

Ahmed Palić¹, Maid Omerović¹ and Edisa Korda²

¹ Edukacijski fakultet, Univerzitet u Travniku, Bosna i Hercegovina

² Osnovna škola "Turbe" u Travniku, Bosna i Hercegovina

Sažetak. U ovom čemu radu istražiti konkretne i neophodne teorijske i praktične osnove koje se odnose na načelo matematičke indukcije i Peanove aksiome, metodologiju istog te razinu znanja studenata. U prvom dijelu rada razmatrat ćemo matematičku indukciju, te pokušati, pomoću nekoliko primjera, pokazati najjednostavnije razumijevanje matematičke indukcije kao i Peanovih aksioma. U drugom dijelu rada predstaviti ćemo metodologiju istraživanja navedenog problema. U trećem dijelu predstaviti ćemo analizu i rezultate istraživanja o metodama matematike i matematičke kompetencije nastavnika matematike, kroz prizmu i rješavanje istih problema. Istraživanje je obuhvatilo studente druge, treće i četvrte godine Edukacijskog fakulteta sa sljedećim sudjelovanjem: studenti druge godine čine 15.4% uzorka, studenti treće godine 23.1% i studenti četvrte godine 6.5%. Studenti dolaze s dvaju odjela Edukacijskog fakulteta Univerziteta u Travniku; 84,6% s Odsjeka za matematiku i informatiku 15,4% s Odsjeka za razrednu nastavu. U istraživanju ćemo koristiti analitičko-deskriptivnu metodu, metodu teorijske analize, uzročnu metodu i metodu istraživanja. Rezultati istraživanja prikazat će se u grafičkom i tabličnom obliku s objašnjenjem i raspravom. Iz ovog rada i njegovog zaključka očekujemo utvrđivanje trenutnog stanja teorijske i praktične osnove principa matematičke indukcije i Peanovih aksioma, kao i razine znanja studenata na ovom području.

Ključne riječi: matematička indukcija, Peanovi aksiomi, kompetencije

Midlines of a quadrilateral

Sead Rešić¹, Fatih Destović² and Nermin Hodžić³

¹ Faculty of Science, Department of Mathematics, University of Tuzla, Bosnia and Herzegovina

² Faculty of Pedagogy in Sarajevo, University of Sarajevo, Bosnia and Herzegovina

³ University of Travnik, Bosnia and Herzegovina

Abstract. By observing the history of geometry, we can see that some of the most attractive problems have been concurrency of three lines and collinearity of three points. Two astonishing results related to these problems are the theorems of Ceva and Menelaus, which are well known to most mathematicians, especially the competitive ones. Here we present a theorem of a quadrilateral, applying these theorems and van Aubel's theorem. Namely, we prove that in the quadrilateral ABCD, whose diagonals meet at O, the midline of a triangle BOC is parallel to BC, the midline of a triangle AOD is parallel to AD, and the midlines of a quadrilateral ABCD connecting the sides AB and CD meet at one point or are parallel.

We also present interesting collinearity and concurrency in a quadrilateral, and another equivalent form of Ceva's theorem, which is also useful. The main point is that these results are derived using only similarity and midlines of triangles, which are well known to primary school students as well as the beginners among high school students. We will consider a number of statements related to the mentioned concepts.

Keywords: Ceva's theorem, Menelaus theorem, cevian, concurrency, collinearity

Srednjice četverokuta

Sead Rešić¹, Fatih Destović² i Nermin Hodžić³

¹ Prirodno-matematički fakultet, Univerzitet u Tuzli, Bosna i Hercegovina

² Pedagoški fakultet, Univerzitet u Sarajevu, Bosna i Hercegovina

³ Univerzitet u Travniku, Bosna i Hercegovina

Sažetak. Neki od najzanimljivijih problema kroz povijest geometrije bili su konkurentnost triju pravaca i kolinearnost triju točaka. Dvije značajna tvrdnje vezane uz ta dva problema su Cevin i Menelajev teorem, koji su većini matematičara dobro poznati, a posebno natjecateljima. Ovdje razmatramo teorem o četverokutu, primjenjujući ova dva teorema i van Aubelov teorem. Naime, ako se u četverokutu ABCD dijagonale sijeku u točki O, tada se sljedeća tri pravca: pravac na kojima leži srednjica trokuta BOC paralelna s BC; pravac na kojem leži srednjica trokuta AOD paralelna s AD i pravac koji prolazi polovištima stranica AB i CD, sijeku u jednoj točki ili su paralelna.

Također predstavljamo zanimljivu kolinearnost i konkurentnost u četverokutu i jedan ekvivalentni oblik Cevinog teorema, koji je također primjenjiv. Važno je napomenuti da su ovi rezultati dobiveni samo pomoću sličnosti i svojstava srednjica trokuta koji su dobro poznati kako učenicima osnovnih škola, tako i početnicima među srednjoškolcima. Bit će promatrane različite tvrdnje koje povezuju spomenute pojmove.

Ključne riječi: Cevin teorem, Menelajev teorem, cevijana, konkurentnost, kolinearnost

The influence of inclusion on the conative and cognitive characteristics of children in mathematics teaching

Sead Rešić¹, Ahmed Palić² and Edisa Korda³

¹ Faculty of Science, Department of Mathematics, University of Tuzla, Bosnia and Herzegovina

² Faculty of Education, University of Travnik, Bosnia and Herzegovina

³ Elementary school “Turbe“ in Travnik, Bosnia and Herzegovina

Abstract. In this research, we analyzed the cognitive and conative characteristics of the eighth-grade students of primary nine-year education such as: motivation for learning mathematics, situational interest in learning mathematics during school classes, mathematical anxiety, self-esteem regarding school achievement, attribute of success and failure in mathematics and the role of assistant in teaching mathematics for children with disabilities. On a sample of 136 eighth-grade students and 17 teachers, we analyzed the cognitive and conative characteristics of the above-achieving students in mathematics, under-achieving students and average students. The research was conducted using the questionnaire and the examination of students' knowledge. The results of the research will be presented in graphical and tabular form with explanation and

discussion. In conclusion we will outline the directions that should be taken to further improve this insufficiently studied area.

Keywords: mathematics, talent, children with developmental disabilities, cognitive and conative qualities, teaching, teacher, student

Utjecaj inkluzije na konativne i kognitivne osobine učenika u nastavi matematike

Sead Rešić¹, Ahmed Palić² i Edisa Korda³

¹ Prirodno-matematički fakultet, Univerzitet u Tuzli, Bosna i Hercegovina

² Edukacijski fakultet, Univerzitet u Travniku, Bosna i Hercegovina

³ Osnovna škola “Turbe“ u Travniku, Bosna i Hercegovina

Sažetak. U ovom istraživanju analizirali smo kognitivne i konativne karakteristike učenika osmog razreda primarnog devetogodišnjeg obrazovanja kao što su: motivacija za učenje matematike, situacijsko zanimanje za učenje matematike tijekom nastave, matematička anksioznost, samopoštovanje u odnosu na školski uspjeh, atribut uspjeha i neuspjeha u matematici i uloga asistenta u nastavi matematike za djecu s teškoćama u razvoju. Na uzorku od 136 učenika osmog razreda i 17 nastavnika analizirali smo kognitivne i konativne karakteristike natprosječnih učenika, ispodprosječnih učenika i prosječnih učenika u postignućima iz matematike. Istraživanje smo proveli koristeći upitnik i test. Rezultati istraživanja bit će prikazani u grafičkom i tabličnom obliku s objašnjenjem i raspravom. U zaključku ističemo smjernice koje treba poduzeti za daljnje poboljšanje ovog nedovoljno istraženog područja.

Ključne riječi: matematika, talent, djeca s teškoćama u razvoju, kognitivne i konativne kvalitete, poučavanje, učitelj, učenik

Problems and problem situation at the teaching topic example "Number divisibility and applications"

Sead Rešić¹, Fatih Destović², Alma Šehanović³ and Amila Osmić⁴

¹ Faculty of Science, Department of Mathematics, University of Tuzla, Bosnia and Herzegovina

² Faculty of Pedagogy in Sarajevo, University of Sarajevo, Bosnia and Herzegovina

³ Gymnasium "Meša Selimović", Tuzla, Bosnia and Herzegovina

⁴ Construction geodesy school, Tuzla, Bosnia and Herzegovina

Abstract. Nowadays one of the most important research direction is exploring of mathematical language knowledge, which presents an ability of the person to understand and recognize the mathematics in everyday life, to create conclusions which are supported by the facts and proofs and to apply mathematics in the way that will respond to a life requests. The generally accepted opinion is that the teaching lesson is not efficient if the students are not working active and by themselves, if they are not solving the problems which request a giving an idea, cleverness and a certain level of creativity. It is confirmed that the use of certain education methods can contribute to the development of students' knowledge. In our country, very few experimental studies are done in the number theory area. Considering the importance of students engagement and the development of students' creative ability, we decided to reconsider educational effects one of the modern educational methods, i.e. problem learning with parallel equal groups in the first grade of general high school and in first grade of the mathematical high school on the topic "Number divisibility and applications".

Keywords: Problem learning, number divisibility, lesson organization, experimental observing teaching effects, research results

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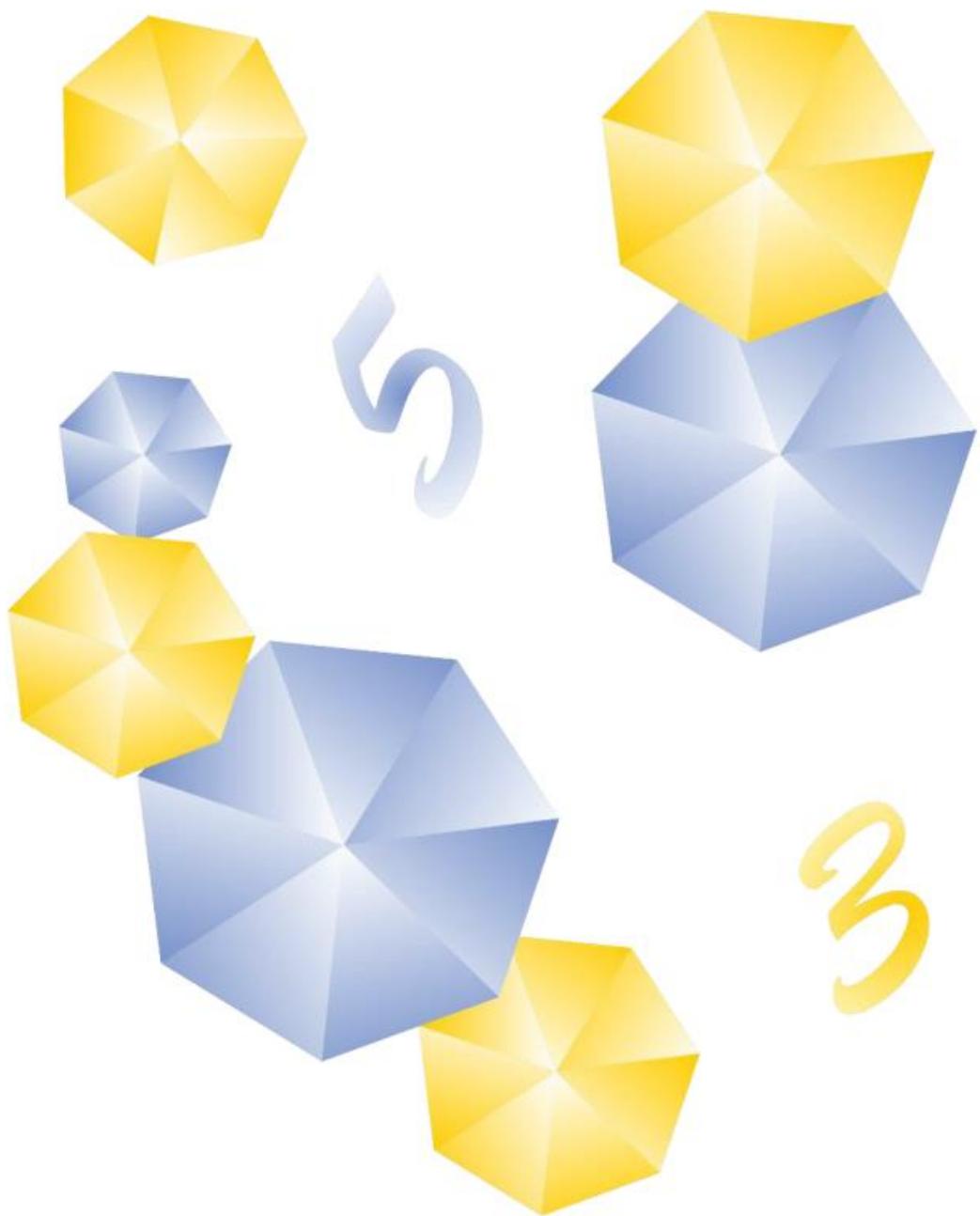
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