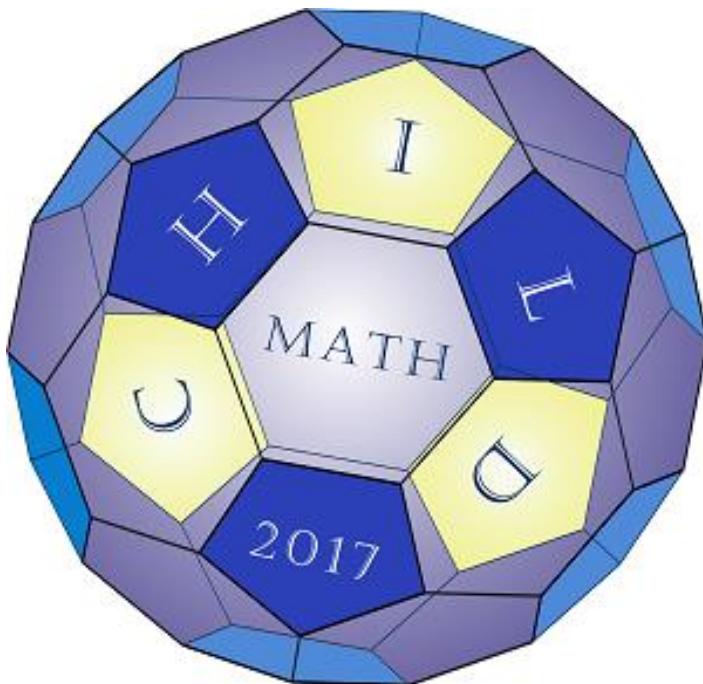




Faculty of Education

Department of Mathematics



**The 6<sup>th</sup> International Scientific Colloquium  
MATHEMATICS AND CHILDREN  
founded by Margita Pavleković**

**PROGRAM  
&  
BOOK OF ABSTRACTS**

**Editors:**  
**Zdenka Kolar-Begović**  
**Ružica Kolar-Šuper**  
**Ljerka Jukić Matić**

**Croatia, Osijek, May 26-27, 2017**

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**Josip Juraj Strossmayer University of Osijek  
Faculty of Education and Department of Mathematics**

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# **PROGRAM**



**FRIDAY, May 26**

8.00 - 9.00 Registration

9.00 - 9.30 **OPENING**

**Morning Session**

<b>Plenary lecture</b>	
9.30-10.15	Bettina Dahl <i>The study approaches of university students in a calculus class</i>
10.15-10.30	Ljerka Jukić Matić, Dubravka Glasnović Gracin <i>Teachers' beliefs on mathematics as a background for their teaching practice</i>
10.30-10.45	Aleksandra Čižmešija, Ana Katalenić, Željka Milin Šipuš <i>Asymptote as a body of knowledge to be taught in textbooks for Croatian secondary education</i>
10.45-11.00	Matea Gusić <i>Functions in the 2015 and 2016 Croatian State Matura in higher level Mathematics</i>
11.00-11.30	Coffee Break

11.30-11.45	Dušan Mundar, Damira Keček <i>Targeting additional effort for students' success improvement: The highest effect group selection method</i>
11.45-12.00	Josipa Matotek <i>Mathematics Attitudes among Students of Civil Engineering</i>
12.00-12.15	Ksenija Romstein, Ljiljana Pintarić Mlinar <i>Teaching and learning mathematics in inclusive settings: Analysis of curriculum of compulsory education in five European countries</i>
12.15-12.30	Juriј Marussig <i>Physical experiments for blind and visually impaired students in elementary school</i>
12.30-12.45	Edith Debrenti <i>Problem solving in elementary mathematics education</i>
12.45-14.30	Lunch Break

### Afternoon Session

	<b>Plenary lecture</b>
14.30-15.15	Željka Milin Šipuš <i>Searching for a common ground in mathematics and physics education: the case of integral</i>
15:15-15.30	Željka Dijanić, Goran Trupčević <i>The impact of using GeoGebra interactive applets on conceptual and procedural knowledge</i>
15.30-15.45	Luka Marohnić, Mandi Orlić Bachler <i>Applications of free computational software in math courses at Zagreb University of Applied Sciences</i>
15.45-16.15	Coffee Break

	Emil Molnár, István Prok, Jenő Szirmai <i>The football {5, 6, 6} and its geometries: from a sport tool to fullerenes and further</i>
16.15-17.00	Nikolina Kovačević <i>Spatial reasoning in mathematics</i>
17.00-17.15	A. S. Leeds, Nataša Macura, Zachary Moring <i>Holes in alien quilts</i>
17.15-17.30	<b>Poster session</b> Melita Štefan Trubić, Vanja Čotić, Sanja Vranić <i>Application of modern technology in teaching mathematics at the Faculty of Engineering</i> Marta Vinković, Ana Katalenić, Zdenka Kolar-Begović <i>An analysis of ten-year old pupils' solutions to a geometrical task with regard to assessment of mathematical giftedness using Math-gift</i>

19.30           Colloquium Dinner (Restoran Vinarija Josić, Zmajevac)

**SATURDAY, May 27**

**Morning Session**

	<b>Plenary lecture</b>
9.00-9.45	Attila Bölcseki <i>What kind of math do you really need to become an architect?</i>
9.45-10.00	Bojan Kovačić, Mirela Katić Žlepalo <i>The use of the computer program Graph in teaching application of differential calculus</i>
10.00-10.15	Goran Trupčević, Anda Valent <i>Language of Croatian Mathematical Textbooks</i>
10.15-10.30	Ljiljanka Kvesić, Anela Čolak <i>Evaluation of mathematical achievements of students in primary education</i>
10.30-11.00	Coffee Break

11.00-11.15	Amanda Glavaš, Azra Staščik <i>Enhancing positive attitude towards mathematics through introducing Escape Room games</i>
11.15-11.30	Zoran Horvat <i>Standardization of learning outcomes in teaching mathematics</i>
11.30-11.45	Ružica Kolar-Šuper, Andrea Sadrić, Zdenka Kolar-Begović, Petra Abičić <i>The presence of mathematical games in primary school</i>
11.45-12.00	Ivana Đurđević Babić, Ana Kozić, Tomislav Milić <i>Discovering student profiles with regard to the use of mathematics tutoring services at university level</i>
12.00-12.15	Ivana Đurđević Babić, Ana Kozić, Tomislav Milić <i>Identifying mathematical anxiety with MLP and RBF neural networks</i>

**12.15 CLOSING**



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## **ABSTRACTS**



## **What kind of math do you really need to become an architect?**

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Attila Bölcseki

Institute of Architecture, Szent István University, Ybl Miklós Faculty of Architecture and Civil Engineering, Budapest, Hungary

*Abstract.* In the presentation we try to provide a complete overview of the higher education of Architects in the field of Mathematics in Hungary. The topic is very actual these days. Last year the Government decided to change the system of training and output requirements of all the BSc and MSc diplomas. Beside smaller ones, the main change concerning Architects is the length of the studies: the Universities have to switch from 8+3 (BSc+MSc) semesters, to 7+4, and the new system must start in September 2017 already.

Although the change could mostly be administrative, the Architects seize the opportunity to radically reform their whole education system. A serious and basic question is whether they do really need Mathematics at all?

In the presentation we give the answer of the competent Universities, and the content of the new Curricula.

We will look more closely the spatial abilities, because it has strong connections to geometrical skills, which are undoubtedly relevant for Architects. In the last few years we measured the spatial perception of our students by different standard tests. We examined the data using statistical methods and hypothesis testing. The results are convincing that descriptive geometry is one of the most efficient tools in developing the spatial abilities.

On the other hand, we collected a lot of experiences during the problem solving process of the students. Based on it, and in order to help the work of the secondary school teachers, recommendations are made, which, if accepted, could make the students more well-prepared for the University requirements. The talk also

contains these advices and the structure of the corresponding secondary school education in Mathematics.

*Keywords:* engineering education, mathematics for architects, standard spatial ability test, secondary school mathematics teaching

## **Milyen matematikai ismeretre is van szüksége egy építésznek?**

Bölcsei Attila

Szent István Egyetem, Ybl Miklós Építéstudományi Kar, Építészmérnöki Intézet, Budapest,  
Hungary

*Absztrakt.* Az előadásban megpróbálkozunk azzal, hogy teljes körképet adjunk a magyar építészmérnök képzésben előforduló matematikai tartalmakról. A téma napjainkban nagyon aktuális. 2016-ban került kiadásra az a kormányrendelet, mely az egyetemi alap és mesterképzésbeli szakok új képzési és kimeneti követelményeit szabályozza. Az építészmérnök képzést illetően a sok apróbb változásból a leglényegesebb különbség a tanulmányok hosszában van: a korábbi 8 féléves BSc és 3 féléves MSc rendszert egy 7+4-es új váltja. Az új rendszerű képzések már 2017 szeptemberétől indulnak.

Habár a változásokat szinte adminisztratív szinten is kezelní lehetne, az építész oktatói társadalom megragadja a lehetőséget, hogy az egész képzést radikálisan megváltoztassa. Alapvető és húsbavágó kérdésként merült fel, hogy szükség van-e egyáltalan matematikára?

Az előadásban az érintett egyetemek által megfogalmazott válaszokat tárgyaljuk, az új tantárgyak tartalmával együtt.

Közelebbről fogjuk vizsgálni a téri intelligencia tárgykörét, mely a geometriai képességekkel mutat szoros kapcsolatot, és amely kétségtelenül releváns az építészek számára. Az utóbbi években több standard teszt segítségével végeztünk felmérést hallgatóink körében a térlátást vizsgálandó. Az adatokat statisztikai eljárásokkal, hipotézis vizsgálattal dolgoztuk fel. Az eredmények arról győznek meg bennünket, hogy az ábrázoló geometria egyike a leghatékonyabb térszemlélet fejlesztő módszereknek.

A hallgatók feladatmegoldásai és a velük való interakció során rengeteg tapasztalatot gyűjtöttünk a matematikai ismeretek szintjéről. Ezeket alapul véve, abból a célból, hogy a középiskolai tanárok munkáját segítsük, ajánlásokat fogalmazunk meg számukra, melyekkel diákjait a műszaki képzés számára jobban

tudnák felkészíteni. Az előadás során a magyar középiskolai matematika képzéssel kapcsolatos információkat és az említett ajánlásokat is megosztjuk a Hallgatósgággal.

*Kulcsszavak:* mérnökképzés, matematika építészeknek, standard térlátás mérő tesztek, középiskolai matematikaoktatás

## **The study approaches of university students in a calculus class**

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Bettina Dahl

Aalborg Centre for Problem Based Learning in Engineering, Science and Sustainability under the auspices of UNESCO, Aalborg University, Denmark

*Abstract.* 191 US first-year university students got the ASSIST (Approaches and Study Skills Inventory for Students) questionnaire as part of a mid-term course evaluation. The students were not in any science, engineering, or mathematics study programme but took a calculus course to satisfy the university breadth requirement for mathematics. The strategic approach was the most commonly used. There was a positive correlation between the deep and the strategic approaches and a negative correlation between the surface and the strategic approaches. There was no correlation between the deep and the surface approaches except a negative correlation between the sub-scales Lack of purpose and Interest in ideas. The surface approach had a negative effect, while the strategic approach had a positive effect on learning outcome.

*Keywords:* learning approaches, study approaches, deep approach, strategic approach, surface approach, calculus, university students, learning outcome

## **Universitetsstuderendes studiestrategier på et calculushold**

Bettina Dahl

Aalborg Centre for Problem Based Learning in Engineering, Science and Sustainability under the auspices of UNESCO, Aalborg Universitet, Danmark

*Abstrakt.* 191 førsteårsstuderende fra USA fik ASSIST (Approaches and Study Skills Inventory for Students) spørgeskemaet som del af en midtvejsevaluering på et kursus. Ingen af de studerende læste til ingeniør eller var indskrevet på naturvidenskab eller matematik, men alle tog calculus for at opfylde universitetets krav om bredde i uddannelse, herunder matematik. Det var den strategiske fremgangsmåde som blev brugt oftest. Der var en positiv korrelation mellem den dybdegående og den strategiske fremgangsmåde og en negativ korrelation mellem den overfladiske og den strategiske fremgangsmåde. Der var ingen korrelation mellem den dybdegående og den strategiske fremgangsmåde på nær på underskalaerne: 'Lack of purpose' og 'Interest in ideas'. Den overfladiske fremgangsmåde havde en negative effekt på læringsudbyttet, mens den strategiske fremgangsmåde have en positive effekt.

*Søgeord:* læringsstil, studiefremgangsmåde, dyb fremgangsmåde, strategisk fremgangsmåde, overfladisk fremgangsmåde, calculus, universitetsstuderende, læringsudbytte

## **Searching for a common ground in mathematics and physics education: the case of integral**

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Željka Milin Šipuš

Faculty of Science, University of Zagreb, Croatia

*Abstract.* In mathematics and physics students are often asked to extract information and give interpretation of the slope (gradient) of a line graph and the area under the graph. However, differences in contexts and context specificities cause difficulties for students to realize that they are studying the same concept (e.g. Leinhardt, Zaslavsky & Stein 1990; McDermott, Rosenquist, & van Zee 1987). The study of Woolnough (2000) reveals the existence of student resistance to apply their mathematical knowledge to physics. It suggests that high school students operate in three distinct contexts: the real world, the physics world and the mathematical world, each with own characteristics and belief systems and without substantial links between them. In our study conducted at the Faculty of Science, University of Zagreb, we have analyzed and compared the first-year university students' strategies and difficulties on mathematically similar problems concerning line graphs situated in mathematics, physics and real world contexts (Ivanjek, Susac, Planinic, Andrasevic & Milin Sipus (2016); Planinic, Susac, Ivanjek & Milin Sipus (2013); Planinic, Milin Sipus, Katic, Susac & Ivanjek (2012)). The analysis was based on the test with eight sets of parallel items from mathematics, physics and other contexts, which was administered to 385 first year students (prospective physics or mathematics teachers, physicists or mathematicians). Three problems in each set were parallel in the sense that they all required the same mathematical procedure to be solved (e.g. computation of a graph slope or the area under a graph). In the presentation we present the summary of the final results of our study with the special emphasis on the mathematical context, in particular on the total change of a quantity, i.e. an integral of a function. Besides, we also bring in focus a common approach in mathematics and physics education – inquiry based teaching and learning and discuss

the third domain of our study and a modelling activities as an introductory common ground.

*Keywords:* interdisciplinary field of mathematics and physics, mathematics education, physics education, inquiry based teaching and learning, slope, area under the graph

## **Određeni integral u nastavi matematike i fizike**

Željka Milin Šipuš

<sup>1</sup>Prirodoslovno-matematički fakultet, Sveučilište u Zagrebu, Hrvatska

*Sažetak.* U matematici i fizici učenici i studenti često su pozvani traženu informaciju dobiti interpretiranjem nagiba grafa linearne funkcije ili površine ispod takvog grafa. Pritom razlike u kontekstu te specifičnosti konteksta studentima često stvaraju poteškoće pri shvaćanju kako proučavaju isti koncept (e.g. Leinhardt, Zaslavsky & Stein 1990; McDermott, Rosenquist, & van Zee 1987). Istraživanje koje je proveo Woolnough (2000) otkriva postojanje svojevrsnog otpora učenika i studenata pri korištenju matematičkog znanja u fizici. Ono sugerira da učenici srednjih škola rade u tri različita konteksta: u kontekstu situacija iz stvarnog svijeta, u fizikalnom kontekstu te matematičkom kontekstu, pri čemu svaki od tih konteksta ima vlastite karakteristike i sustave vjerovanja, a značajnije međusobne veze ne postoje. U našem istraživanju provedenom na Prirodoslovno-matematičkom fakultetu Sveučilišta u Zagrebu analizirali smo i uspoređivali strategije studenata prve godine studija matematike i fizike, kao i njihove poteškoće pri rješavanju matematički sličnih zadataka s grafovima linearnih funkcija koji su smješteni u matematički, fizikalni ili stvarni kontekst (Ivanjek, Sušac, Planinić, Andrašević & Milin Šipuš (2016); Planinić, Sušac, Ivanjek & Milin Šipuš (2013); Planinić, Milin Šipuš, Katić, Sušac & Ivanjek (2012)). Istraživanje se temeljilo na upitniku od osam kompleta paralelnih pitanja u matematičkom, fizikalnom i stvarnom kontekstu. U njemu je sudjelovalo 385 studenata prve godine (nastavničkih smjerova matematike ili fizike, te budućih matematičara ili fizičara). Tri pitanja u svakom kompletu bila su paralelna u smislu da su zahtijevala pri rješavanju isti matematički postupak (npr. izračunavanje nagiba grafa ili površine ispod grafa).

U ovom predavanju prikazujemo sažetak konačnih rezultata naše studije s posebnim naglaskom na matematički kontekst, te posebice rezultate o ukupnoj promjeni veličine, tj. određenom integralu funkcije. Pored toga, razmatramo zajednički pristup u nastavi matematike i fizike – istraživački usmjereno učenje i

poučavanje, te raspravljamo o trećem kontekstu naše studije, kontekstu stvarnih situacija, i aktivnostima modeliranja kao zajedničkom polju djelovanja u nastavi.

*Ključne riječi:* interdisciplinarnost, nastava matematike, nastava fizike, istraživački usmjerena nastava, nagib, površina ispod grafa

## **Asymptote as a body of knowledge to be taught in textbooks for Croatian secondary education**

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Aleksandra Čižmešija<sup>1</sup>, Ana Katalenić<sup>2</sup> and Željka Milin Šipuš<sup>1</sup>

<sup>1</sup>Faculty of Science, University of Zagreb, Croatia

<sup>2</sup>Faculty of Education, University of Osijek, Croatia

*Abstract.* Asymptote and asymptotic behaviour are bodies of knowledge present in elementary as well as in theoretical and applied mathematics. Some aspects of these concepts make a common part of upper secondary mathematics curricula worldwide. The notion of an asymptote appears as a theoretical concept, a part of a procedure or as a self-sufficient task in different contexts and different educational cycles of upper secondary mathematics education.

In this paper, we present results of a textbook analysis on asymptote and asymptotic behaviour in two most common series of gymnasium mathematics textbooks in Croatia. It is a part of a larger comprehensive study regarding this body of knowledge in general upper secondary education in Croatia. The research is conducted within the theoretical framework of the Anthropological theory of the didactics (ATD), developed by a French mathematician Y. Chevallard. This framework provides praxeology  $[T, \tau, \theta, \Theta]$  as a tool for describing mathematical knowledge and activities, with its practical component – a task  $T$  and a technique  $\tau$ , and discursive or theoretical component – a technology  $\theta$  and a theory  $\Theta$ .

The results show that the asymptote is a complex body of knowledge consisted of several praxeologies for which it is a part of practical or theoretical component. Hence, we present the related praxeological organization of considered textbooks along with its direct implications to mathematics teaching and learning.

*Keywords:* asymptote, asymptotic behaviour, ATD, praxeology, textbook analysis, general secondary mathematics education

## **Asimptota kao objekt znanja za poučavanje u udžbenicima za srednju školu u Hrvatskoj**

Aleksandra Čižmešija<sup>1</sup>, Ana Katalenić<sup>2</sup> i Željka Milin Šipuš<sup>1</sup>

<sup>1</sup>Prirodoslovno-matematički fakultet, Sveučilište u Zagrebu, Hrvatska

<sup>2</sup>Fakultet za odgojne i obrazovne znanosti, Sveučilište u Osijeku, Hrvatska

*Sažetak.* Asimptota i asimptotsko ponašanje objekti su matematičkog znanja prisutni u elementarnoj, teorijskoj i primjenjenoj matematici. Neki aspekti tih koncepata dijelovi su srednjoškolskog matematičkog kurikuluma diljem svijeta. Pojam asimptote se javlja kao teorijski koncept, dio tehnike ili zaseban zadatak te u više obrazovnih razdoblja i u različitim nastavnim sadržajima predmeta matematike.

U ovom radu prikazujemo rezultate analize udžbenika s obzirom na asimptotu i asimptotsko ponašanje u dva najčešće korištena seta udžbenika za gimnazijalno matematičko obrazovanje. Provedena analiza je dio opsežnog istraživanja vezanog uz taj objekt znanja u srednjoškolskom obrazovanju u Republici Hrvatskoj. Teorijski okvir unutar kojega je istraživanje provedeno jest antropološka teorija didaktike (ATD), koju je razvio francuski matematičar Y. Chevallard. U tom je teorijskom okviru prakseologija  $[T, \tau, \theta, \Theta]$  model kojim se opisuju matematičko znanje i aktivnosti, koji ima praktičnu komponentu – zadatak  $T$  i tehniku  $\tau$ , te diskurzivnu ili teorijsku komponentu – tehnologiju  $\theta$  i teoriju  $\Theta$ .

Rezultati pokazuju kako je asimptota složen objekt znanja koji se sastoji od više prakseologija kojima je ona dijelom praktične ili teorijske komponente. Prikazat ćemo s time povezanu prakseološku organizaciju odabranih udžbenika i odgovarajuće implikacije na učenje i poučavanje matematike.

*Ključne riječi:* asimptota, asimptotsko ponašanje, ATD, prakseologija, analiza udžbenika, srednjoškolsko matematičko obrazovanje

## **Problem solving in elementary mathematics education**

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Edith Debrenti

Faculty of Economics and Social Sciences, Partium Christian University, Oradea, Romania

*Abstract.* General problem-solving skills are of central importance in mathematics achievement. Problem-solving strategies and cognitive operations should be taught in the classroom consciously. In most cases, the focus is on the problem-solving process, rather than the result. How do we solve problems? There are different models for this. Pólya's model develops reasoning and divides problem solving into four steps (Pólya, 2000).

Strategy keys can be an alternative way to develop students' problem solving capacity by heuristics (Herold- Blasius, Jazby, 2016).

The present paper presents an experiment involving third and fourth grade (9-10 years old) students who have been asked to solve certain mathematical problems. Two experimental groups were involved. The aim was to investigate whether problem solving is more efficient when using the traditional method (working on paper with pencil) or when using keys. We hypothesized that using keys might prove helpful in solving problems.

*Keywords:* problem-solving skills, unusual problems, heuristics, strategy keys, mathematical model

## **Problémamegoldás az elemi matematikatanításban**

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*Kivonat.* A problémamegoldó képesség általános fejlesztése fontos célkitűzés a matematikatanításban. A probléma-megoldási stratégiákat, gondolkodási műveleteket tudatosan kell tanítani a tanórákon. A legtöbb esetben a problémamegoldási folyamat áll a középpontban, nem pedig a kapott eredmény. A feladatok tanórai feldolgozásának legfontosabb lépései Pólya modellje szerint javasolt, ez a modell a feladatok megoldását négy nagy lépésre bontja. (Pólya, 2000).

A kulcsos problémamegoldási modell egy lehetőség lehet a tanulók feladatmegoldási képességeinek fejlesztésére. (Herold- Blasius, Jazby, 2016).

Ebben a kétképes klasszikus pedagógiai kutatásban azt vizsgáltuk, hogy ha feladatmegoldási stratégiákkal ismertetjük meg a tanulókat, ha szöveges feladatok megoldására különböző módszereket, eljárásokat, modelleket tanítunk meg a harmadik, illetve negyedik osztályban (9-10 éves gyerekeknek), akkor javul-e a tanulók feladatmegoldási és problémamegoldási képessége? Feltételeztük, hogy a kulcsok használata elősegíti a probléma-megoldási képességet.

*Kulcsszavak:* probléma megoldási képesség, szöveges feladatok, heurisztika, kulcsos stratégiák, matematikai modell

## **The impact of using GeoGebra interactive applets on conceptual and procedural knowledge**

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*Abstract.* This work deals with the impact of using digital textbooks consisting of GeoGebra interactive applets on students' conceptual and procedural knowledge in mathematics. The authors propose the model of computer-guided discovery learning by using dynamic geometry software which includes three basic elements: learning objects, tasks for students and learning outcomes. The model is primary based on the theory of constructivism, Pólya's heuristic strategy and Schoenfeld's problem-solving model.

The experimental research plan with experimental and control groups was used and there were 703 students in twelve elementary schools in Croatia participating in the research. The experimental group of students was taught by the model of computer-guided discovery learning and the control group of students was taught by teachers using traditional teaching methods. The covariance analysis determined the impact of the model of computer-guided discovery learning on students' conceptual and procedural knowledge and the results have shown statistically significant differences between the groups in favor of the experimental group.

The qualitative and quantitative analysis of students' and teachers' questionnaires and forum discussions was used to assess their attitudes towards the proposed learning model. The importance of teachers' experience in teaching using this model has also been noted. This learning model indicates the potential of computers and dynamic geometry software GeoGebra for scaffolding support in learning mathematics.

*Keywords:* conceptual knowledge, digital textbook, discovery learning, dynamic geometry software, GeoGebra, interactive applet, learning model, procedural knowledge

## **Utjecaj uporabe interaktivnih apleta kreiranih u GeoGebri na konceptualno i proceduralno znanje učenika**

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*Sažetak.* Ovaj se rad bavi utjecajem uporabe digitalnih udžbenika koji sadrže interaktivne aplete kreirane u GeoGebri na konceptualno i proceduralno znanje učenika. Autori predlažu model računalno vođenoga učenja otkrivanjem korištenjem programa dinamične geometrije koji uključuje tri osnovna elementa: objekte učenja, aktivnosti za učenike i ishode učenja. Model se temelji na teoriji konstruktivizma, Pólyinoj heurističkoj strategiji i Schoenfeldovom modelu rješavanja problema.

U istraživanju je sudjelovalo 703 učenika iz dvanaest osnovnih škola u Hrvatskoj te je korišten eksperimentalni nacrt s eksperimentalnom i kontrolnom skupinom. Eksperimentalna skupina učenika učila je po modelu računalno vođenoga učenja otkrivanjem, a kontrolna skupina učenika u tradicionalno organiziranoj nastavi. Analizom kovarijance utvrđen je utjecaj modela računalno vođenoga učenja otkrivanjem na konceptualno i proceduralno znanje učenika, a rezultati pokazuju statistički značajne razlike između skupina u korist učenika eksperimentalne skupine.

Provedena je kvalitativna i kvantitativna analiza upitnika za učenike i učitelje te analiza rasprave učitelja na forumu kako bi se ustanovili stavovi učenika i učitelja prema predloženome modelu učenja. Uočena je važnost iskustva učitelja u primjeni ovoga modela učenja. Predloženi model učenja ukazuje na potencijal računala i programa dinamične geometrije GeoGebra kao scaffolding podrške u učenju matematike.

*Ključne riječi:* digitalni udžbenik, GeoGebra, interaktivni aplet, konceptualno znanje, model učenja, proceduralno znanje, program dinamične geometrije, učenje otkrivanjem



## **Discovering student profiles with regard to the use of mathematics tutoring services at university level**

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Ivana Đurđević Babić, Ana Kozić and Tomislav Milić

Faculty of Education, University of Osijek, Croatia

*Abstract.* One could argue that there is a growing trend in Croatian society of using tutoring services from different subjects at all levels of education. This research, using classification tree approach, deals with the use of mathematics tutoring services at university level and focuses on building student profiles related with its use. Along with questions that provided some general information about students, scores obtained by the Willingness to Listen Measure (Richmond & Hickson, 2001) and Teacher Apprehension Test (Richmond, Wrench & Gorham, 2001) as well as statements describing students' attitudes towards mathematics tutoring were used for classification trees modelling. Several classification tree models were built and from the most successful one student profiles were extracted.

*Keywords:* mathematics tutoring, classification tree, university level, mathematics performance, students

## **Otkrivanje profila studenata s obzirom na korištenje instrukcija iz matematike kod sveučilišnog obrazovanja**

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Fakultet za odgojne i obrazovne znanosti, Sveučilište u Osijeku, Hrvatska

*Sažetak.* Moglo bi se reći da je rastući trend u hrvatskom društvu korištenje instrukcija iz raznih predmeta na svim razinama obrazovanja. Koristeći pristup klasifikacijskih stabala, ovaj rad bavi se upotrebom instrukcija iz matematike na razini sveučilišnog obrazovanja i fokusiran je na izgradnju profila studenata koji koriste takve instrukcije. Zajedno s pitanjima kojima su dobiveni neki opći podatci o studentima, rezultati skala Willingness to Listen (Richmond & Hickson, 2001) i Teacher Apprehension (Richmond, Wrench & Gorham, 2001) kao i izjave koje opisuju stav studenata prema instrukcijama iz matematike, korišteni su kao varijable za modeliranje klasifikacijskih stabala. Izrađeno je nekoliko modela klasifikacijskih stabala te su iz najuspješnijeg izvučeni traženi profili studenata.

*Ključne riječi:* instrukcije iz matematike, klasifikacijsko stablo, sveučilišno obrazovanje, uspješnost iz matematike, studenti

## **Identifying mathematical anxiety with MLP and RBF neural networks**

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Ivana Đurđević Babić, Tomislav Milić and Ana Kozić

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*Abstract.* This research addresses the problem of mathematical anxiety which is usually associated with inadequate mathematical performance and achievement. It aims to develop neural network model for classification of students according to the degree of mathematical anxiety in order to examine and better understand the relationship and effects of physical activity along with some other factors on mathematical anxiety. For this purpose, Multilayer Perceptron (MLP) and Radial Basis Function (RBF) neural networks were used. The results of this research showed that neural network models were efficient in identifying students' mathematical anxiety. With the purpose of exploring the relationships between the mathematical anxiety and input variables, sensitivity analysis was conducted and reported for the model with the highest overall classification accuracy.

*Keywords:* mathematical anxiety, physical activity, neural network, MLP, RBF

## **Identificiranje matematičke anksioznosti uz pomoć MLP i RBF neuronskih mreža**

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*Abstract.* U ovom istraživanju razmatra se problem matematičke anksioznosti koja je obično povezana s nedostatnim postignućima i neodgovarajućim uspjehom iz matematike. Istraživanje je usmjereni na razvijanje modela neuronske mreže u svrhu klasifikacije studenata prema stupnju matematičke anksioznosti s namjerom da se istraži i bolje razumije povezanost i utjecaj tjelesne aktivnosti i nekih drugih čimbenika na matematičku anksioznost. U tu svrhu korištene su mreže višeslojni perceptron (engl. multilayer perceptron, MLP) i mreža s radikalno zasnovanom funkcijom (engl. radial basis function, RBF). Rezultati ovog istraživanja pokazali su da su modeli neuronskih mreža bili učinkoviti u prepoznavanju matematičke anksioznosti kod studenata. Za model s kojim se postigla najviša ukupna točnost klasifikacije provedena je analiza osjetljivosti kako bi se istražila povezanost između matematičke anksioznosti i ulaznih varijabli.

*Keywords:* matematička anksioznost, tjelesna aktivnost, neuronske mreže, MLP, RBF

## **Functions in the 2015 and 2016 Croatian State Matura in higher level Mathematics**

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Matea Gusić

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*Abstract.* This paper presents an analysis of Croatian State Matura exam tasks relating to the mathematical domain of functions. The analysis is conducted on the 2015 and the 2016 State Matura, both higher-level mathematics exams. Requirements of the tasks are explored and classified based on the instrument designed for this purpose. The instrument is conceptually based on the theory of “basic ideas” (originally: Grundvorstellungen) and Mathematical Proficiency theory. The objectives of the research were to explore the possibility of using such an instrument for task analysis, to determine the characteristics of the tasks in the Croatian State Matura, and to use student achievement to value their efficiency in the mathematical domain of functions in terms of specific classification provided by the instrument.

*Keywords:* state matura, Grundvorstellungen, mathematical proficiency, function, task analysis

## **Funkcije na višoj razini hrvatske Državne mature provedene 2015. i 2016. godine**

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*Sažetak.* Rad prikazuje analizu zadataka iz područja funkcija, na višoj razini hrvatske Državne mature, provedene na ljetnom ispitnom roku 2015. i 2016. godine. Analizi zadataka prethodi uvođenje teoretskog okvira na temelju kojeg su se klasificirali zahtjevi zadataka iz područja funkcija. Razvijeni teoretski okvir baziran je na teoriji „osnovnih ideja“ (Grundvorstellungen) i „matematičkih vještina“ (Mathematical Proficiency). Ciljevi rada bili su istraživanje mogućnosti korištenja ovako definiranog teorijskog okvira kao kriterija za analizu i određivanje karakteristika zadataka iz područja funkcija na hrvatskoj Državnoj maturi te mogućnosti evaluacije specifičnih sposobnosti iz područja funkcija na temelju uspješnosti rješavanja zadataka i njihove klasifikacije prema definiranom teoretskom okviru.

*Ključne riječi:* državna matura, Grundvorstellungen, matematičke vještine, funkcije, analiza zadataka

## **Standardization of learning outcomes in teaching mathematics**

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Zoran Horvat

Electro technical School Varaždin, Croatia

*Abstract.* This work presents critical approach in analysis of recent curriculum documents and literature relevant for evaluation of learning outcomes. Author focuses on advantages and disadvantages of standardization using didactical and pedagogical approach to mathematics education. In recent literature related for teaching mathematics there is growing focus on social and pedagogical dimensions of teaching and learning as key assumptions of students' success in mathematics. Therefore, learning outcomes should evaluate all parts of mathematics curriculum, but in national curriculum documents only educational outcomes of mathematics content are being relevant. Author indicates negative implications of neglecting pedagogical dimensions of teaching and learning mathematics, and emphasizes contradiction between standardization and pedagogical evaluation of learning outcomes. Considering the totality of factors influencing mathematics education, validation of learning outcomes should include pedagogical and psychological dimensions of teaching process and emotional and affective components of learning outcomes. In the conclusion, author suggests guidelines toward upgrading learning outcomes by integrating students' abilities, needs, work habits, attitudes and beliefs with clear objectives, goal setting methods and assessments specified in the curriculum.

*Keywords:* learning outcomes, social and affective dimensions of teaching and learning mathematics

## **Standardizacija obrazovnih ishoda u nastavi matematike**

Zoran Horvat

Elektrostrojarska škola Varaždin, Hrvatska

*Sažetak.* Autor u ovom radu kritički analizira recentne kurikularne dokumente i literaturu relevantnu za evaluaciju (odgojno)-obrazovnih ishoda. Naglasak je pritom na prednostima i nedostacima standardizacije sa didaktičkog i pedagoškog aspekta matematičkog obrazovanja. U recentnoj literaturi učenja i poučavanja matematike sve je veći naglasak na socijalnim i pedagoškim dimenzijama nastave matematike kao ključnim prepostavkama uspjeha učenika. U skladu s time obrazovni ishodi trebaju evaluirati sve komponentne kurikuluma matematike, no u aktualnim kurikularnim dokumentima relevantni su jedino standardi obrazovnih postignuća. Autor ističe negativne posljedice zapostavljanja odgojnih dimenzija nastave matematike te ističe proturječja između standardizacije i pedagoške evaluacije učeničkih postignuća. Uvažavajući sveukupnost čimbenika koji utječu na matematičko obrazovanje, konceptualizacija obrazovnih ishoda mora sadržavati pedagoško-psihološku dimenziju nastavnog procesa, te emocionalne i afektivne komponente učeničkih postignuća. Zaključno, autor predlaže smjernice za nadogradnju obrazovnih ishoda integracijom stavova, sposobnosti, radnih navika i uvjerenja učenika sa ishodima, metodama i evaluacijom zadanim u kurikulumu.

*Ključne riječi:* obrazovni ishodi, socijalne i afektivne dimenzije nastave matematike

## **Teachers' beliefs on mathematics as a background for their teaching practice**

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Ljerka Jukić Matić<sup>1</sup> and Dubravka Glasnović Gracin<sup>2</sup>

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*Abstract.* This paper discusses the impact that teachers' beliefs on mathematics and mathematics education have on their teaching practice. The issues concerning teaching practice focus in particular on the utilization of mathematics textbooks in the classroom. It is argued that teachers' beliefs about teaching and learning mathematics are significant in the utilization of a particular resource. We conducted a case study involving lower secondary mathematics teachers (grades five to eight) where we examined teachers' classroom practice and beliefs about mathematics, mathematics education, teaching mathematics and using textbooks as curriculum resources. The aim was to find out whether the teachers' attitudes correspond to their practice in the observed lessons or whether there are inconsistencies. The findings suggest that teachers' decisions regarding the use of textbooks in mathematics are influenced by their perception of the educational value of the textbooks and their personal beliefs about mathematics and mathematics education.

*Keywords:* mathematics teacher, beliefs, textbook, teacher-curriculum relationship

## **Učiteljeva uvjerenja o matematici kao pozadina njihove nastavne prakse**

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*Sažetak.* U ovom radu promatra se utjecaj učiteljevih uvjerenja i stavova o matematici i nastavi matematike na njihovu nastavnu praksu. Pitanje nastavne prakse posebno se usredotočuje na upotrebu udžbenika u nastavi matematike.

Učiteljevi stavovi i uvjerenja o učenju i poučavanju matematike važni su kod upotrebe određenog kurikularnog materijala tj. nastavnog sredstva. Proveli smo studiju slučaja koja uključuje učitelje nastave matematike u osnovnoj školi. Istraživanje smo proveli pomoću opservacija i intervjeta koji su se fokusirali na učiteljeva uvjerenja o matematici, matematičkom obrazovanju, podučavanju matematike i korištenju udžbenika kao kurikularnog materijala. Cilj je bio utvrditi odgovaraju li stavovi promatranih učitelja njihovoj nastavnoj praksi ili postoje nedosljednosti. Rezultati upućuju na zaključak da na odluku o tome treba li rabiti udžbenik u nastavi ili ne, utječe percipirana obrazovna vrijednost udžbenika kao i osobna uvjerenja učitelja o matematici i matematičkom obrazovanju.

*Ključne riječi:* učitelj matematike, uvjerenja, kurikularna sredstva, odnos učitelj-kurikulum

## **The presence of mathematical games in primary school**

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Ružica Kolar-Šuper<sup>1</sup>, Andrea Sadrić<sup>2</sup>, Zdenka Kolar-Begović<sup>3</sup> and Petra Abičić<sup>1</sup>

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*Abstract.* One of the most important goals of teaching mathematics that needs to be taken into account is to stimulate interest in and love for mathematics. A number of studies have confirmed that the introduction of different types of educational games in mathematics education increases student interest in mathematics and has a positive impact on student achievement in different fields of mathematics. Through well-designed games that are implemented correctly in mathematics classes students acquire knowledge in a new and more interesting way and develop a positive attitude towards mathematics, which represents the greatest value and one of the basic goals of teaching mathematics. The aim of this paper is to investigate the frequency of using mathematical games in mathematics revision classes in the lower grades of primary school (the first cycle) and to learn about the types of games teachers use when teaching mathematics, including computer games in mathematics classes.

*Keywords:* mathematical game, computer game, motivation, mathematics teaching, mathematics education

## **Prisutnost matematičkih igara u osnovnoj školi**

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*Sažetak.* Jedan od najvažnijih ciljeva nastave matematike o kojem je nužno voditi računa je izazivanje interesa i pobuđivanje ljubavi prema matematičkim sadržajima. Nizom istraživanja potvrđeno je da uvođenja različitih vrsta didaktičkih igara na nastavi matematike povećava kod učenika interes prema matematici te ima pozitivan utjecaj na postignuća učenika iz različitih područja matematike. Kroz dobro osmišljene i provedene igru u nastavi matematike učenici uče na nov i zanimljiviji način te formiraju pozitivan stav prema matematici, što predstavlja najveću vrijednost i jedan od osnovnih ciljeva nastave matematike. Cilj ovog rada je istražiti učestalost primjene matematičkih igara pri ponavljanju matematičkih sadržaja u nižim razredima osnovne škole (prvom odgojno obrazovnom ciklusu) te dobiti saznanja o vrstama igara koje učitelji koriste u nastavi matematike uključujući i računalne igre na satu matematike.

*Ključne riječi:* matematička igra, računalna igra, motivacija, nastava matematike, matematičko obrazovanje

## **Spatial reasoning in mathematics**

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Nikolina Kovačević

Faculty of Mining, Geology and Petroleum Engineering, Department of Mathematics, Informatics and Descriptive Geometry, University of Zagreb, Croatia

*Abstract.* With the growing interest in spatial reasoning, stimulated by the development of powerful computer-based geometry and visualization packages, it is important to be clear about what is meant by spatial reasoning in mathematics. Starting from the point of various math educators, learning spatial thinking in mathematics has different aims than learning spatial thinking in other sciences.

Hence, although spatial skills may be intellectually interesting in themselves, the focus in this paper is placed on its relationship with teaching and learning geometry at the technical faculties. Furthermore, the course Descriptive geometry with computer graphics, which has evolved at the Faculty of Mining, Geology and Petroleum Engineering in Zagreb in conjunction with the recent developments in the modern geometry education, is described in detail. On the basis of the classical geometrical representation methods, the course focuses not only on the uprising of graphic-visual communication and developing learners' spatial visualization skills, which play a crucial role in engineering educations, but likewise on the development of learners' capacity with deductive reasoning and making use of aids and tools in mathematics education. Also, the effect of computer technology on geometry education is discussed according to the results of the SEFI - Mathematics Working Group (SEFI - stands for "European Society for Engineering Education"). The examples of student exercises will be given to show a large range of options offered within the course to make teaching of space mathematics innovating, more interactive and at the same time applicable to specific students' interests.

*Keywords:* spatial reasoning, teaching tools, computer graphics, higher education, e-learning

## **Prostorno rasuđivanje u matematici**

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*Sažetak.* Uz sve veći interes za prostornim rasuđivanjem, potaknut razvojem snažnih vizualizacijskih programa i računalne geometrije, važno je razjasniti što se podrazumijeva pod prostornim rasuđivanjem u matematici. Polazeći od gledišta različitih matematičkih edukatora, ostvarenje istog obrazovnog cilja razvijanje prostornog rasuđivanja ne ostvaruje se uvijek na isti način u matematici kao i u drugim znanostima.

Dakle, premda prostorne sposobnosti mogu biti same po sebi intelektualno zanimljive, u ovom se radu fokus stavlja na njihovu povezanost s poučavanjem i učenjem geometrije na tehničkim fakultetima. Nadalje, detaljno će se opisati kolegij Nacrtna geometrija s računalnom grafikom koji je nastao na Rudarsko-geološko-naftnom fakultetu u Zagrebu slijedeći suvremene trendove razvoja geometrijskog obrazovanja. Koristeći između ostalog i tradicionalne geometrijske metode reprezentacije, kolegij se ne usmjerava samo na podizanje razine grafičke i vizualne komunikacije i razvijanje prostornih sposobnosti pojedinca, koje imaju ključnu ulogu u obrazovanju inženjera, već i na razvoj sposobnosti deduktivnog rasuđivanja te korištenje različitih alata i pomagala u matematičkom obrazovanju inženjera. U radu se raspravlja i o utjecaju računalne tehnologije na geometrijsko obrazovanje slijedeći smjernice SEFI - matematičke radne skupine. Dani su i različiti primjeri studentskih vježbi kako bi se prikazale brojne mogućnosti koje se nude studentima kroz razvoj inovativnih i interaktivnih obrazovnih metoda istovremeno primjenjivih u učenju matematike prostora i usklađenih sa specifičnim studentskim interesima.

*Ključne riječi:* prostorno rasuđivanje, nastavna pomagala, računalna grafika, visoko obrazovanje, e-učenje

## **The use of the computer program *Graph* in teaching application of differential calculus**

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*Abstract.* The paper presents the authors' own teaching experience in the application of the computer program *Graph* in teaching unit *Problem of determining the tangent line and the normal line to the graph of real functions of one variable* within the mathematical courses in professional study programs of Zagreb University of Applied Sciences. The typical problems encountered during teaching the aforementioned unit and concrete examples how these problems can be successfully solved methodically are presented.

*Keywords:* computer program Graph, differential calculus, tangent line, normal line, graphs of functions of one variable

## **Primjena računalnoga programa *Graph* u poučavanju primjene diferencijalnoga računa**

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*Sažetak.* U radu se izlažu vlastita nastavna iskustva autora u primjeni računalnoga programa *Graph* u obradi nastavne cjeline *Problem određivanja tangente i normale na graf realne funkcije jedne realne varijable* u sastavu matematičkih predmeta na stručnim studijima Tehničkoga veleučilišta u Zagrebu. Navode se tipični problemi uočeni u obradi navedene nastavne cjeline, te na konkretnim primjerima objašnjava kako se ti problemi metodički mogu uspješno riješiti.

*Ključne riječi:* računalni program *Graph*, diferencijalni račun, tangenta, normala, grafovi funkcija jedne varijable.

## **Evaluation of mathematical achievements of students in primary education**

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*Abstract.* The main problem in training students for their work in primary education is to develop criteria by which students will master the sufficient level of knowledge in mathematics necessary for development of logical thinking and reasoning required for other sciences and life by itself. Thus it is necessary to develop in students certain level of knowledge in the field of mathematics in order that future teachers can be able to transfer that knowledge on students. The question is what is the level of the knowledge that must be achieved by future teachers? The coincidence in the assessment of knowledge is avoided and objectives of educational process are met through summative and formative valuation. In this paper we will try to analyze the results of summative and formative valuation of the students of primary education from the course Mathematics 1 and Mathematics 2 at the Faculty of Science and Education, University of Mostar in order that teaching goals contribute to the quality of teaching mathematics in primary education.

*Keywords:* teaching goals, teaching in primary education, mathematics, formative and summative valuation, level of knowledge according to standards

## **Vrednovanje postignuća studenata u nastavi matematike u primarnom obrazovanju**

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*Sažetak.* Temeljni problem u osposobljavanju studenata za rad u primarnom obrazovanju je razviti kriterij po kojem će studenti savladati dovoljnu razinu znanja iz matematike koja je potreba za razvijanje logičkog mišljenja i zaključivanja potrebnog za druge znanosti i sam život. Samim time potrebno je kod studenata razviti određenu razinu znanja iz područja matematike kako bi budući učitelji mogli to znanje prenijeti na učenike. Postavlja se pitanje koja je to razina znanja koju budući učitelji moraju postići? Kroz sumativno i formativno vrednovanje izbjegava se slučajnost u ocjeni znanja i zadovoljavaju ciljevi odgojno–obrazovnog procesa. Kroz ovaj rad pokušati će se analizirati rezultati sumativnog i formativnog vrednovanja studenata primarnog obrazovanja iz kolegija Matematika 1 i Matematika 2 na Fakultetu prirodoslovno-matematičkih i odgojnih znanosti, Sveučilišta u Mostaru kako bi ishodi poučavanja doprinijeli kvaliteti nastave matematike u primarnom obrazovanju.

*Ključne riječi:* ishodi učenja, nastava u primarnom obrazovanju, matematika, formativno sumativno vrednovanje, razina znanja po standardima

## ***Holes in alien quilts***

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*Abstract.* Mathematical developments in last two decades brought cube complexes in a spotlight of the research in theoretical mathematics. The concept of CAT(0)cube complexes emerged as a particularly elegant structure suitable for modeling a variety of problems. We will start with the definition of cube complex and then introduce the large link and extra large link conditions, two combinatorial properties of CAT(0) cube complexes. Using examples of two-dimensional CAT(0) cube complexes, we will explain how these properties affect the geometry of the complex, in particular, the divergence and the isoperimetric inequality.

*Keywords:* CAT(0) spaces, growth, divergence, isoperimetric inequality



## **Applications of free computational software in math courses at Zagreb University of Applied Sciences**

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*Abstract.* In this paper, authors describe their own teaching experience using Maxima and GNU Octave software packages as computational tools in math courses at studies for Electrical and Civil Engineering of Zagreb University of Applied Sciences and discuss possibilities of achieving the same learning outcomes using software package Giac/Xcas. Finally, a way of modernizing math courses by introduction and usage of suitable computer software is proposed, emphasizing the importance of using free software in education.

*Keywords:* teaching mathematics, free software, computer algebra system, Maxima, Octave, Giac/Xcas

## **Primjena slobodnih softvera u nastavi matematike na Tehničkom veleučilištu u Zagrebu**

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*Sažetak.* U radu autori opisuju svoja iskustva u primjeni programskih paketa Maxima i GNU Octave u nastavi matematike na stručnim studijima elektrotehnike i graditeljstva na Tehničkom veleučilištu u Zagrebu, te ukazuju na mogućnosti postizanja istih ishoda učenja primjenom softverskog paketa Giac/Xcas. Na kraju rada, predlažu način modernizacije nastave matematike, uvođenjem i korištenjem odgovarajućeg računalnog softvera, naglašavajući važnost korištenja besplatnog softvera u obrazovanju.

*Ključne riječi:* podučavanje matematike, slobodni softver, računalni algebarski sustav, Maxima, Octave, Giac/Xcas

## **Physical experiments for blind and visually impaired students in elementary school**

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*Abstract.* This article presents the problem of independent prosecution physical experiments, which are actively involved blind and visually impaired students. An inclusive school must also blind and visually impaired students providing the best possible integration into the learning process, which means as big as possible independent in their learning as well as in the prosecution of physical experiments, while the rest of the students can learn a lot from blind and visually impaired classmates. This is the essential moment of inclusion process. Autonomy and active cooperation is the most important part of personal motivation blind in visually impaired student. In this article are present some physical experiments about light and its nature. Experiments are performed by blind and visually impaired students from Institution of Blind and Visually Impaired youth in Ljubljana. The goal of independent perform physical experiments is more qualitatively performance blind and visually impaired students about some physical problem. When the students learned about phisysical experiments they were been active and motivated for work. That means the experiments with light were interesting and understandable. Blind and visually impaired students were being independent, didactic accessories for them were being enough understandable for processing their mind activity. We found out that the blind and visually impaired students got a lot of knowledge about experiments with light, which are adapted for them. With production adjusted physical experiments for blind and visually impaired students we were get closer blind and visually impaired students needs for learning physics.

*Keywords:* blind and visually impaired students, adapted physical experiments, a beam of light, inclusion

## **Prilagoditve fizikalnih eksperimentov z elektriko za slepe in slabovidne učence**

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*Abstraktna.* Uvajanje inkluzije v osnovno šolo, torej sprejemanje otrok s posebnimi potrebami v redne osnovne šole, je temelj družbe prihodnosti, saj imamo vsi različni med seboj enake pravice do izobraževanja. Tako se proces inkluzije počasi vgraje v celoten vzgojno-izobraževalni sistem, v kurikulum, in zato tudi v vse učne ure vsakega predmeta. Uvajanje inkluzije je sočasno povezano s strokovnostjo učitelja ter njegovim sprejemanjem in dojemanjem različnosti učencev preko njegovih empatičnih osebnostnih značilnosti. Potrebno je dobro poznavanje posebnih potreb učencev in metod dela z njimi pri vsaki učni uri skozi ves vzgojno-izobraževalni proces. To velja tudi za slepe in slabovidne učence, ki se v Sloveniji številčno čedadje bolj vključujejo v redne izobraževalne procese. V raziskavi o prilagoditvah pouka slepim in slabovidnim učencem smo se osredotočili na pouk fizike in fizikalne eksperimente s ciljem, da bodo slepi in slabovidni učenci pri izvajanju eksperimentov čim bolj samostojni in da bomo v razredu vzpostavili čim kakovostnejšo inkluzivno klimo.

Eno od najobširnejših poglavij v fiziki v devetem razredu je poglavje o električnem toku, naboju in električni napetosti. Poglavlje je v večini zasnovano eksperimentalno, saj učenci s pomočjo izkustvenega učenja ter eksperimenti z električnim tokom pridobivajo znanje in poglobljene predstave o pojavi elektrike. Eksperimenti so zasnovani na podlagi majhnih električnih tokov in so povezani z delovanjem porabnikov, kot so žarnice.

Ker je zaznavanje svetlobe za slepe in slabovidne učence zelo težavno, so nujne kakovostne prilagoditve, ki svetlobo žarnic zamenjajo s tipnim zaznavanjem, ter prilagoditve pri drugih elektronskih elementih. Rezultati raziskave kažejo napredek v doseganju višjih Bloomovih taksonomskeih stopenj v znanju slepih in slabovidnih učencev.

*Ključne besede:* inkluzija, otroci s posebnimi potrebami, slepi in slabovidni otroci, fizikalni eksperimenti v osnovni šoli

## **Mathematics Attitudes among Students of Civil Engineering**

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*Abstract.* The aim of this paper is to identify attitudes of undergraduate students of Civil Engineering towards mathematics by studying the responses of 193 students. This study examines whether there is a connection between attitudes towards mathematics and performance on exams in mathematics. Furthermore, differences in attitudes about math regarding gender and prior education (vocational or grammar school) were also analyzed. The Attitude Toward Mathematics Inventory (ATMI) questionnaire was used for this research. Data were analyzed with analytics software package Statistica 13, t-test, Mann-Whitney and Kruskal-Wallis tests were also used.

The survey results suggest a conclusion that there is no difference in attitudes towards mathematics regarding gender. Grammar school students scored significantly higher than vocational students in overall attitudes, but also in all four sub-scales Enjoyment, Motivation, Value and Self-confidence. Students with the lowest scores on the exam have more negative attitudes and vice versa. This also goes for the subscales Enjoyment, Value and Self-confidence, but not for Motivation.

*Keywords:* math attitudes, math performance, ATMI questionnaire, statistical analysis, undergraduate students

## **Stavovi prema matematici studenata Građevinskog fakulteta**

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*Sažetak.* Cilj ovoga rada je identificirati stavove prema matematici studenata Građevinskog fakulteta analizirajući odgovore 193 studenta. U ovom članku se ispituje postoji li veza između stavova prema matematici i uspješnosti u polaganju ispita iz matematike. Također, analizirane su razlike u stavovima prema matematici s obzirom na spol i prijašnje obrazovanje (stručna srednja škola ili gimnazija). Za prikupljanje podataka je korišten upitnik ATMI. Podaci su analizirani uz pomoć programskega paketa Statistica 13, u tu svrhu su korišteni sljedeći testovi: t-test, Mann-Whitney i Kruskal-Wallis test.

Iz rezultat istraživanja zaključujemo da nema razlike u stavovima prema matematici u odnosu na spol. Studenti koji su završili neku od gimnazija su postigli značajno više rezultate od studenata koji su završili neku od stručnih srednjih škola u ukupnim stavovima, ali i u sve četiri kategorije pitanja: zadovoljstvo, motivacija, vrijednost i samopouzdanje. Pokazano je da studenti s najnižim ocjenama na ispitu imaju najlošije (najnegativnije) ukupne stavove prema matematici, ali i obrnuto. Isto vrijedi i za kategorije pitanja zadovoljstvo, vrijednost i samopouzdanje, ali ne i za motivaciju.

*Ključne riječi:* stavovi prema matematici, uspješnost u matematici, upitnik ATMI, statistička analiza, prediplomski studenti

## **The football {5, 6, 6} and its geometries: from a sport tool to fullerenes and further**

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Emil Molnár, István Prok and Jenő Szirmai

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*Abstract.* This presentation starts with the regular polygons, of course, then with the Platonic and Archimedean solids. The latter ones are whose symmetry groups are transitive on the vertices, and in addition, whose faces are regular polygons (see only I. Prok's home page [1] for them). Then there come these symmetry groups themselves (starting with the cube and octahedron, of course, then icosahedron and dodecahedron). Then come the space filling properties: Namely the cube is a space filler for the Euclidean space  $E^3$ . Then we jump for the other regular solids that cannot fill  $E^3$ , but can hyperbolic space  $H^3$ , a new space. These can be understood better if we start regular polygons, of course, that cannot fill  $E^2$  in general, but can fill the new plane  $H^2$ , as hyperbolic or Bolyai-Lobachevsky plane. Now it raises the problem, whether a football polyhedron - with its congruent copies - fill a space. It turns out that  $E^3$  is excluded (it remains an open problem – for you, of course, in other aspects), but  $H^3$  can be filled as a schematic construction show this (Figure 1), far from elementary. Then we mention some stories on Buckminster Fuller, an architect, who imagined first time fullerenes as such crystal structures. Many problems remain open, of course, we are just in the middle of living science.

*Keywords:* Mathematics teacher as popularizer of science, Platonic and Archimedean solid, tiling, Euclidean and non-Euclidean manifold, crystal structure, fullerenes

[1] Prok, I. (2014). <http://www.math.bme.hu/~prok/> [Accessed 05/08/2014]

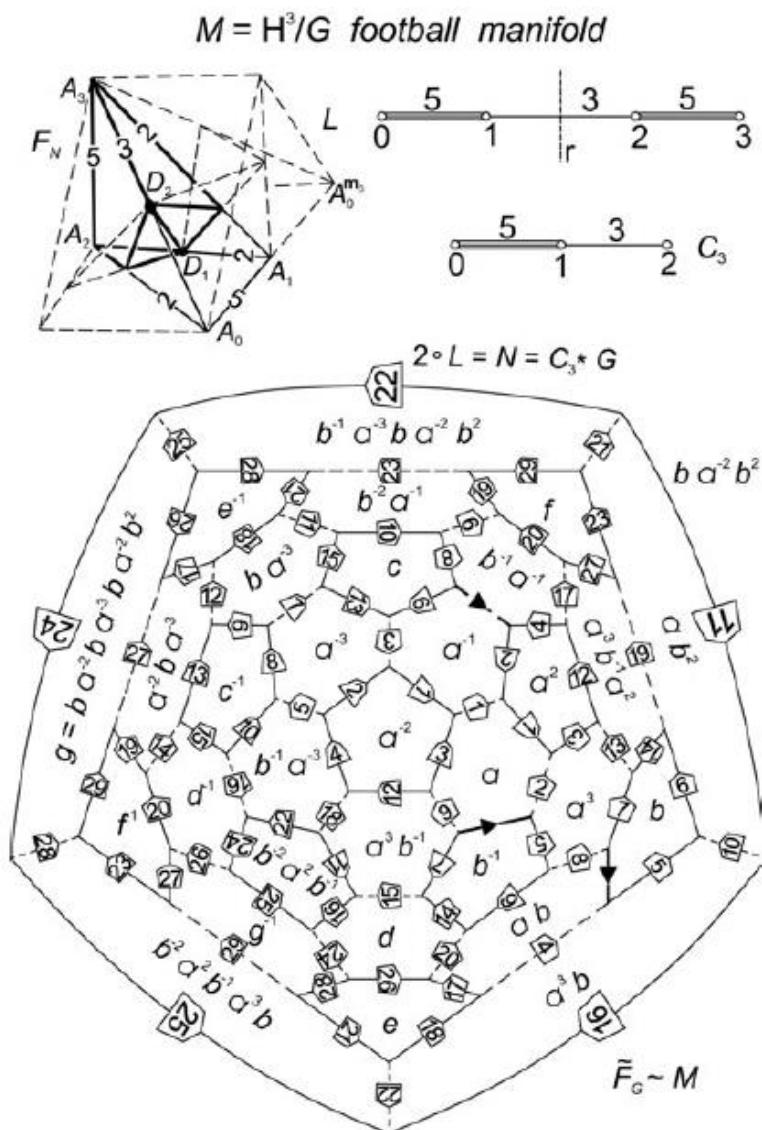


Figure 1. The hyperbolic football manifold for the Archimedean solid  $\{5, 6, 6\}$ .

## Az {5, 6, 6} futball és geometriája: a játékszertől a fullerénekig és tovább

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*Osszefoglaló.* Ez a “tudomány-népszerűsítő” előadás a szabályos sokszögekből indul ki. Aztán Prok I. honlapját bemutatva, a Platón-féle szabályos és az Archimédesz-féle fél szabályos testekkel, ezek szimmetriáival folytatja. A gömbi geometria síktükrözései és az úgy-nevezett alaptartományok elemzése lesz a fő eszközünk. Például a kocka középpontjában  $\pi/4 = 45^\circ$ ,  $\pi/3 = 60^\circ$  és  $\pi/2 = 90^\circ$  lapszögű szimmetriasíkok találkoznak. Ezekre a síkokra tükrözve, ennek a karakterisztikus gömbháromszögnek 48 példánya kövezi ki a gömböt, amit úgy is mondunk: a kocka szimmetriacoportja 48 elemű és 3 síktükrözés generálja. A többi szabályos és fél szabályos testet, sőt síkbeli szabályos mintákat is jellemzhetünk így. A karakterisztikus háromszög szögösszegének a  $\pi = 180^\circ$ -tól való eltérése jellemzi a szabályos testeket, ha az eltérés pozitív; a kockánál pl.  $\pi/3 + \pi/4 + \pi/2 - \pi = \pi/12$ . Az euklideszi mintáknál ez a szögkülönbség 0. De elképzelhető negatív eltérés is, ha a háromszög szögösszege kisebb  $180^\circ$ -nál. Ez jellemzi a Bolyai-Lobacsevszkij-féle hiperbolikus sík mintáit, kövezésein. A tér kövezései esetében is fontos szerepet játszanak a tükrözések, melyeket a lineáris algebra eszközeivel terjeszthetünk ki L. Schläfli és H.S.M. Coxeter nyomán.

Kiderül, hogy az {5, 6, 6} szimbólumú futball-labda, melynél minden csúcsban egy szabályos (gömbi) ötszög és két szabályos (gömbi) hatszög találkozik, poliéder - azaz síklapú test - formájában, egybevágó példányaival nem tudja kitölteni euklideszi terünket, de a Bolyai Lobacsevszkij-féle hiperbolikus teret ki tudja tölteni (ahogy ezt Molnár E. 1988-ban észrevette és publikálta. Sőt a kitöltés fixpont-mentes egybevágóságokkal történhet: A futball-labdák olyanok, mintha egy véges téren lennének, ahol minden pontnak hiperbolikus labda környezete van. Úgy tűnik, hogy az anyag tudományokban előterbe került fullerének, azaz  $C_{60}$  molekulák ezt a nagyon

„sűrű hiperbolikus anyagot” követik (melynek további szélső-érték tulajdonságai Szirmai J. és Molnár E. kutatásaiban is előjönnek).

Tehát Bolyai János és N. I. Lobacsevszkij hiperbolikus geometriájának kristálytani alkalmazásai is lehetnek az eddig csillagméretű vonatkozások mellett.

*Kulcsszavak:* A matematika tanár, mint a tudomány népszerűsítője; platóni és archimédeszi test; kövezés; euklideszi és nem euklideszi sokaság; kristálystruktúra, fullerén

## **Targeting additional effort for students' success improvement: The highest effect group selection method**

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*Abstract.* Goal of our research is to present a method for selection of a group of students. The selected group of students is supposed to receive additional teaching attention in order to improve their performance in the course. Guiding line used, for the group selection, is: value of an action is proportional to the benefit that produces to a customer. In our case customers are students. The selection method is based on multinomial logistic regression, Poisson-type discrete variables modelling the number of points achieved in a term exam, and transition probability matrix. We use data of a study progress during monitoring program in an undergraduate mathematical course. Demographic data and other attributes about previous performance were not included in the analyses. In the first part of the paper we present methodology, while in the second part we introduce data used for demonstration of the proposed method. At the end of the paper, an individual approach for the final selection is proposed. Criteria for selection is clear: increased probabilities of obtaining desired final grade for a student. Weighing of criteria is subjective depending on the goals of the decision makers.

*Keywords:* learning analytics, predictive analytics, students' success, multinomial logistic regression, Poisson distribution

## **Usmjeravanje dodatnog angažmana za poboljšanje uspjeha studenata: odabir grupe s najvećim učinkom**

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*Sažetak.* Cilj našeg rada je predstaviti jednu metodu za odabir grupe studenata. Odabranoj grupi studenata trebao bi biti usmjeren dodatni nastavnički angažman kako bi studenti te grupe popravili uspjeh na predmetu. Princip vrednovanja uspješnosti odabira grupe je: vrijednost poduhvata proporcionalna je koristi koju poduhvat stvara kod korisnika. U našem slučaju korisnici su studenti, a korist je povećana vjerojatnost uspjeha studenata. Metoda odabira bazira se na multinomijalnoj logističkoj regresiji, modeliranju broja bodova ostvarenih na kolokviju diskretnim slučajnim varijablama Poissonovog tipa i vjerojatnosnim matricama prijelaza. U radu su korišteni podaci s kontinuiranog praćenja napretka studenata na matematičkom predmetu preddiplomskog studija. Demografski podaci o studentima, kao ni podaci o uspjehu studenata na prethodnim predmetima, nisu korišteni u analizi. U prvom dijelu predstavljena je metodologija, a u drugom dijelu predstavljaju se podatci koji se koriste za demonstraciju metode. Na kraju rada predstavljamo mogućnost individualnog pristupa konačnom odabiru grupe. Kriteriji za odabir su jednoznačni: povećane vjerojatnosti ostvarenja željene završne ocjene studenta. Težine kriterija su subjektivne i ovise o ciljevima donositelja odluke.

*Ključne riječi:* analitika učenja, prediktivna analitika, uspjeh studenata, multinomijalna logistička regresija, Poissonova distribucija

**Teaching and learning mathematics in inclusive settings:  
Analysis of curriculum of compulsory education  
in five European countries**

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*Abstract.* Global perspectives on inclusive education advocate one's effective social functioning. In that perspective, math education is a priority due to its presence in everyday life. Independent living presupposes presence of basic math skills. To identify the presence of math skills that supports independence (i.e. functional mathematical skills), content analysis of five European countries national curriculums was conducted: Great Britain, Finland, Germany, France and Croatia. Results showed that mathematical skills promoted in these national agendas are inherent part of independent living. According to these agendas, functional math can be interpreted as math fluency, numeracy skills, and connection between science and everyday life. Math curriculum is focused on personal experiences and possibilities to achieve basic knowledge and understandings of mathematics, applicable in everyday life. To achieve this goal, some curriculum accommodations should be present, such as applying special teaching strategies, adequate equipment, and facilitate experiential learning.

*Keywords:* inclusive math curriculum, inclusive settings, pupils with disability, functional mathematics

## **Učenje i poučavanje matematike u inkluzivnom okruženju: analiza kurikuluma obveznog odgoja i obrazovanja u pet europskih zemalja**

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*Sažetak.* Globalne perspektive o inkluzivnom obrazovanju stavlju naglasak na djelotovo učinkovito socijalno funkcioniranje. U toj perspektivi, matematika je prioritet, ponajprije zbog njezine prisutnosti u svakodnevnom životu. Samostalno življenje zahtjeva posjedovanje temeljnih matematičkih vještina. Kako bi se identificirale matematičke vještine koje mogu podržavati pojedinčevu samostalnost (tj. funkcionalnih matematičkih vještina), načinjena je analiza sadržaja pet europskih nacionalnih kurikuluma: Velike Britanije, Finske, Njemačke, Francuske i Hrvatske. Rezultati pokazuju kako su matematičke vještine u navedenim dokumentima sastavnice samostalnog življjenja. Prema njima, funkcionalna matematika se može interpretirati kao matematička fluentnost, vještine mjerjenja i brojenja, te povezivanje znanosti i svakodnevnog života. Kurikulumi matematike su usmjereni na osobna iskustva i osiguravanje mogućnosti za jačanje temeljnog matematičkog znanja i razumijevanja matematike, s primjenom u svakodnevnom životu. Kako bi se to postiglo potrebno je načiniti određene kurikulumske prilagodbe, kao što su primjena specifičnih strategija poučavanja, primjerena opremljenost učionica, te podržavanje iskustvenog učenja.

*Ključne riječi:* inkluzivni kurikulum matematike, inkluzivno okruženje, učenici s teškoćama u razvoju, funkcionalna matematika

## **Enhancing positive attitude towards mathematics through introducing Escape Room games**

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*Abstract.* By researching the attitudes towards mathematics, research agreed that these are important factors that appear in the analysis of variation in students' accomplishments in mathematics. Various authors have interpreted the attitudes towards mathematics as an emotional disposition. Moreover, Ma (1997) has shown the proportional correlation between positive attitudes and achievements in mathematics. It is very important for teachers to bare this piece of information in mind and act accordingly. It is considered that positive relation towards mathematics is transmitted by choosing appropriate and innovative methods of teaching and the increase of intrinsic motivation of students. The teachers should be up to date with the contemporary and realistic world, the development of technology, they should follow the students' affinities and according to that create the school lessons. In the past two years, real-life escape room games have achieved a great level of popularity in Croatia. Escape Room games require from participants (in groups of 2 to 5) team work, tolerance towards others, acceptance of diversities for solving logical tasks and problematical situations, while applying wide spectre of knowledge and common culture all under the time pressure. In this paper we investigate MathEscape as relevant method to revise or systematize mathematical (Quadratic equation) content as a method that could affect pupils' attitudes towards mathematics. MathEscape combines technology and teachers' involvement and creativity in creating interesting and real on mathematical content. The research contributes to better understanding of using technology and innovative methods to enhance positive attitude towards mathematics. We also want to encourage teachers to integrate technology and hands on, interesting activities into their teaching since it would help their pupils increase intrinsic motivation. We demonstrate practical example of using Escape Room

games as basis for such type of lessons and activities in order to give teachers some starting point for some more ideas on how to make mathematical content more appealing in order to create positive attitudes.

*Keywords:* attitude, teaching methods, escape room games, math lesson, applying knowledge

## **Poticanje razvoja pozitivnih stavova prema matematici kroz primjenu Escape Room igara**

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*Sažetak.* Istražujući stavove prema matematici, istraživači su se složili kako su stavovi relevantan faktor koji utječe na ostvarenje matematičkih postignuća. Različiti autori, stavove prema matematici smatraju emocionalnom dispozicijom. Ma (1997) ukazuje na proporcionalnu korelaciju pozitivnih stavova i matematičkih postignuća. Vrlo je važno da učitelji te informacije osvijeste i nastavu prilagode tim saznanjima. Smatra se, kako se pozitivan odnos prema matematici prenosi odabirom primjerenih i inovativnih metoda poučavanja kao i podupiranjem razvoja intrinzične motivacije učenika. Strogo se savjetuje da učitelji pri stvaranju plana i programa svoje nastave budu upoznati i prate razvoj suvremene tehnologije, i svakodnevnice te imaju na umu afinitete učenika. U posljednje dvije godine, real-life escape room igre dosegle su veliku razinu popularnosti u Hrvatskoj. Escape Room igre podrazumijevaju timski rad dionika (u grupama od 2 do 5), međusobnu toleranciju i prihvatanje različitosti kako bi riješili logičke zadatke s kojima su suočeni u različitim problemskim situacijama. Od dionika se također traži primjena širokog spektra znanja opće kulture i sve to pod pritiskom vremenskog roka. U ovom istraživanju, ispitali smo MathEscape kao relevantnu metodu vježbanja i sistematizacije matematičkog (Kvadratne jednadžbe) sadržaja koja bi mogla utjecati na razvoj pozitivnih stavova prema matematici. MathEscape kombinira tehnologiju, učiteljev doprinos i kreativnost pri stvaranju zanimljivog i realističnog matematičkog sadržaja. Istraživanje doprinosi dubljem razumijevanju utjecaja, uporabe tehnologije i inovativnih metoda, na poticanje pozitivnih stavova prema matematici. Također, cilj nam je potaknuti učitelje na uključivanje tehnologije te zanimljivih fizičkih aktivnosti u poučavanje matematike kako bi pomogli svojim učenicima razviti intrinzičnu motivaciju za učenje matematike. Predstavili smo i praktičan primjer

Escape Room igre kao podlogu razvoja takvog tipa nastave i nastavnih aktivnosti. Ponuđeni primjer sam je početna točka učiteljima za razvoj vlastitih ideja kojima će matematički sadržaj učiniti privlačnijim kako bi stvorili pozitivne stavove kod svojih učenika.

*Ključne riječi:* stavovi, metode poučavanja, escape room igre, nastava matematike, primjena znanja

## **Language of Croatian Mathematical Textbooks**

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*Abstract.* Mathematical textbooks, as part of a potentially implemented curriculum have been increasingly analysed, mostly from the point of their structural features, performance expectations and mathematical content. In the previous work, we have performed such kind of analysis of Croatian mathematical textbooks for the 4th grade of secondary schools. During this process, the question of language used in the books came out.

In this paper, we use the framework proposed by Lisa O'Keefe and John O'Donoghue (2014), based on the Halliday's functional grammar analysis (Halliday (1985), Halliday and Martin (1993), Morgan (1998)), to perform a preliminary analysis of two Croatian mathematical textbooks.

*Keywords:* functional grammar analysis, mathematics language analysis, Croatian mathematical textbook

## **Jezik u hrvatskim matematičkim udžbenicima**

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*Sažetak.* Matematički udžbenici, kao dio potencijalno implementiranog kurikuluma se analiziraju u sve većoj mjeri, uglavnom obzirom na njihove strukturne karakteristike, očekivanja izvedbe i matematičke sadržaje. U prethodnom radu, na taj smo način analizirali hrvatske matematičke udžbenike za 4. razred gimnazije. Tijekom analize, nametnulo nam se pitanje jezika korištenog u udžbenicima.

U ovom radu koristimo okvir predložen od Lise O’Keefe i Johna O’Donoghuea (2014), koji se bazira na Hallidayevoj funkcionalnoj gramatici (Halliday (1985), Halliday and Martin (1993), Morgan (1998)), kako bi proveli preliminarnu jezičnu analizu dva hrvatska matematička udžbenika.

*Ključne riječi:* funkcionalna gramatika, analiza matematičkog jezika, hrvatski matematički udžbenici

## **Application of modern technology in teaching mathematics at the Faculty of Engineering**

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*Abstract.* Technology is slowly entering the curriculum of mathematics in Croatian primary and secondary schools, and a little faster at the faculty level. This entry often depends on the teacher. Mathematical education at universities has witnessed daily progress, not only because of the introduction of e-learning systems (Moodle, MudRi) but also because of the availability of different technologies and computer programs, which are used for improving teaching, such as Mathematica, Matlab, GeoGebra, and the like. Experiences of teachers show that overcoming certain teaching units can be successfully and easily improved by implementing modern technology in their teaching. The result of this is additional student motivation for adopting mathematical knowledge by connecting various disciplines. Observing the selected problem through mathematical as well as other sciences' perspectives, such as physics, chemistry or biology, has an impact on building a more well-rounded individual who will be able to handle today's challenges. In this way, understanding mathematics as a science, which is a purpose to itself, can be overcome. This in turn contributes to its successful popularization and creates a positive atmosphere about this science as an important tool, which can help in resolving many interdisciplinary problems. The aim of this paper is to present the way in which mathematics can be improved in higher education with the help of selected examples.

*Keywords:* teaching experience, popularization of science, applied mathematics, Mathematica, GeoGebra

## **Primjena suvremene tehnologije u nastavi matematike na Tehničkom fakultetu**

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*Sažetak.* Tehnologija polako ulazi u programe matematike u hrvatskim osnovnim i srednjim školama, i nešto brže na fakultete. Ključnu ulogu u tome ima profesor. Matematičko obrazovanje na sveučilištima svjedoči svakodnevnom napretku, ne samo zbog uvodenja sustava za e-učenje (Moodle, Mudri), već i zbog dostupnosti različitih tehnologija i informatičkih programa koji se mogu koristiti za unapređenje nastave kao što su Mathematica, Matlab, Geogebra, i slično. Iskustava nastavnika pokazuju da se usvajanje nekih nastavnih cjelina može uspješno i jednostavno unaprijediti uvođenjem suvremene tehnologije u nastavu. Rezultat toga je dodatna motivacija učenika za usvajanje matematičkih znanja kroz povezivanje različitih disciplina. Promatranje odabranog problema kroz oči matematike, ali istovremeno i drugih znanosti, kao što su fizika, kemija ili biologija, utječe na izgradnju svestranije osobe koja će se znati nositi s izazovima današnjice. Na taj se način može premostiti mišljenje o matematici kao znanosti koja je sama sebi svrhom, doprinijeti njezinoj uspješnoj popularizaciji i stvoriti pozitivno ozračje o toj nauci kao važnom alatu koji može pomoći u rješavanju mnogih interdisciplinarnih problema. Cilj ovog rada je na odabranim primjerima pokazati kako se nastava matematike može unaprijediti u visokom školstvu.

*Ključne riječi:* iskustvo poučavanja, popularizacija znanosti, primjenjena matematika, Mathematica, GeoGebra

## **An analysis of ten-year old pupils' solutions to a geometrical task with regard to assessment of mathematical giftedness using Math-gift**

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*Abstract.* Recognizing and supporting gifted pupils is a responsibility of the educational system as it is stated in Croatian national curriculum. In 2003. Margita Pavleković started an extracurricular activity named The Little School of Mathematics for ten-year old pupils with special interest in mathematics at Faculty of Education in Osijek. Students, in their final year of teacher studies, work with these pupils as a part of the elective course Mathematics and gifted pupils.

Pavleković, Zekić-Sušac and Đurđević created a neural-network based expert system for recognizing mathematical gift in ten-year old pupils named Math-gift. The expert system places a pupil in either of four categories: (1) potentially gifted child in mathematics, (2) child with mathematical competencies above the average, (3) child with average mathematical competencies or (4) child with mathematical competencies below the average.

In this poster we will present pupils' solution of one geometrical task with regard to the evaluated mathematical giftedness. The results showed a mismatch between the assessment made by students and expert system and indicated that pupils ranked to higher categories tended to solve the task better than the pupils ranked to lower categories. The reasons of the mismatch and means to improve students' assessment and the ways geometrical tasks indicate mathematical giftedness should be further investigated.

*Keywords:* gifted pupils, mathematical giftedness, extracurricular activity, teacher studies, Math-gift

## **Analiza rješenja jednog geometrijskog zadatka u kontekstu procjene matematičke darovitosti desetogodišnjih učenika pomoću Mat-dara**

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*Sažetak.* Prepoznavanje i podrška darovitim učenicima je odgovornost obrazovnog sustava kako stoji u Nacionalnom okvirnom kurikulumu. Margita Pavleković je 2003. godine utemeljila Malu matematičku školu kao izvanškolsku aktivnost za desetogodišnje učenike s posebnim interesom za matematiku pri Fakultetu za odgojne i obrazovne znanosti u Osijeku. Studenti završne godine učiteljskog studija rade s učenicima u sklopu nastave izbornog kolegija Matematika i nadareni učenici.

Pavleković, Zekić-Sušac i Đurđević su izradili ekspertni sustav Mat-dar za prepoznavanje matematičke darovitosti kod desetogodišnjih učenika. Ekspertni sustav svrstava učenika u jednu od četiri kategorije: (1) potencijalno daroviti učenik, (2) učenik iznadprosječnih matematičkih sposobnosti, (3) učenik prosječnih matematičkih sposobnosti ili (4) učenik s nedovoljno razvijenim sposobnostima za matematiku.

Prikazat ćemo učenička rješenja jednog geometrijskog zadatka u kontekstu procjene matematičke darovitosti učenika. Rezultati su pokazali odstupanja u procjeni studenata i ekspertnog sustava te kako su učenici svrstani u više kategorije uglavnom bolje rješili zadatak nego učenici svrstani u niže kategorije. Potrebno je detaljnije istražiti razloge odstupanja i načine kako poboljšati procjene studenata te kako rješavanje geometrijskih zadataka može ukazati na matematičku darovitost učenika.

*Ključne riječi:* daroviti učenici, matematička darovitost, izvanškolska aktivnost, obrazovanje učitelja, Mat-dar

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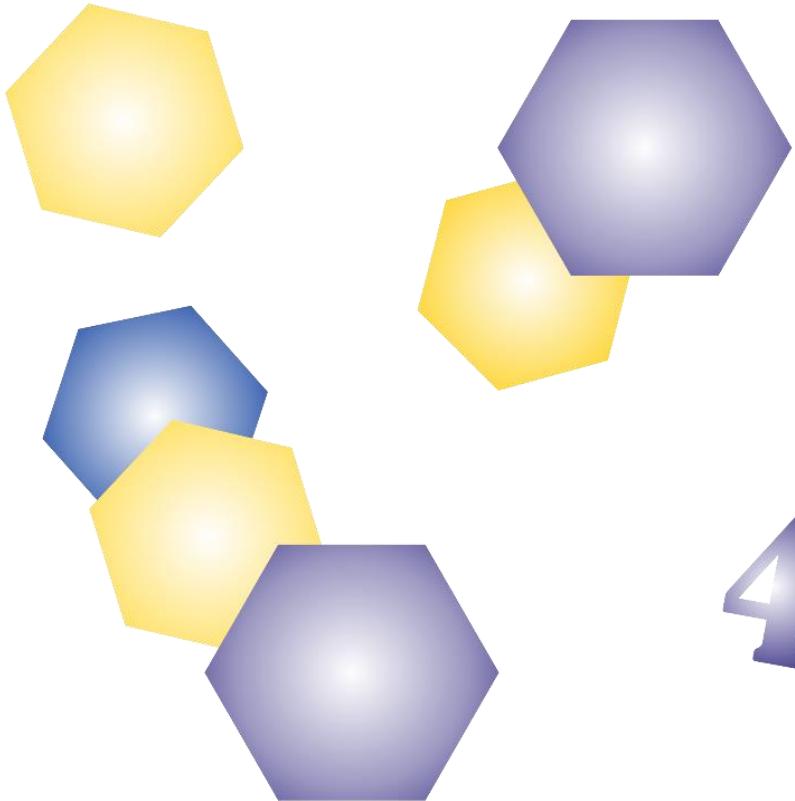
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