Josip Juraj Strossmayer University of Osijek
Faculty of Teacher Education and Department of Mathematics

The 4th International Scientific Colloquium
MATHEMATICS AND CHILDREN

PROGRAM
&
BOOK OF ABSTRACTS

Editors:
Margita Pavleković
Zdenka Kolar-Begović
Ružica Kolar-Šuper

Croatia, Osijek, April 19-20, 2013

Organizer:

Faculty of Teacher Education, Josip Juraj Strossmayer University of Osijek
Department of Mathematics, Josip Juraj Strossmayer University of Osijek

Scientific Committee:

Margita Pavleković
Rudolf Scitovski
Mirta Benšić
Zdenka Kolar-Begović
Ružica Kolar-Šuper
Sibe Mardešić
Šime Ungar
Mara Cotič
Radoslav Galić
Željka Milin Šipuš
Marijana Zekić-Sušac
Blaženka Divjak
Mirko Polonijo
Vladimir Volenec
Emil Molnàr
Éva Kopasz

Josip Juraj Strossmayer University of Osijek
Faculty of Teacher Education and Department of Mathematics

The 4th International Scientific Colloquium
MATHEMATICS AND CHILDREN

PROGRAM
&
BOOK OF ABSTRACTS

Editors:
Margita Pavleković
Zdenka Kolar-Begović
Ružica Kolar-Šuper

Croatia, Osijek
April 19-20, 2013

PROGRAM

FRIDAY, APRIL 19

8.30-9.40 Registration

9.40-10.00 OPENING

Morning session

Chairperson: Zdenka Kolar-Begović

10.00-10.30	Plenary lecture Markus Alexander Helmerich: *Reflected actionability – acquisition of reflection competences for prospective teachers*	*Germany*
10.30-10.45	Dubravka Glasnović Gracin: *Regional comparison of the PISA 2009 results in the field of mathematical literacy*	*Croatia*
10.45-11.00	Maja Škrbec: *Levels of geometric thinking in the second triad of elementary school*	*Slovenia*
11.00-11.15	Maja Cindrić, Irena Mišurac Zorica: *Didactic model of development of multiplicative concept in mathematics in elementary school*	*Croatia*
11.15-11.30	Coffee Break	

Chairperson: Ljerka Jukić Matić

11.30-11.45	Vida Manfreda Kolar, Tatjana Hodnik Čadež: *Dependence of the problem solving strategies on the problem context*	*Slovenia*
11.45-12.00	Miljenko Stanić: *Formal language of mathematics and logic in elementary school instruction of mathematics*	*Croatia*
12.00-12.15	Višnja Vekić Kljaić: *Center for mathematics in living room of the pre-school education institution*	*Croatia*
12.15-12.30	Katalin Munkácsy: *Visualization of mathematical concepts – some examples from geometry teaching*	*Hungary*
12.30-12.45	Goran Lapat, Lidija Eret: *Application of the elements of Vedic mathematics in classes with Roma pupils*	*Croatia*
12.45-13.00	Coffee Break	

Chairperson: Ivana Đurđević

13.00-13.15	Vedrana Mikulić Crnković, Sanja Rukavina: *On the usage of interactive whiteboards in the teaching of mathematics in secondary schools in Primorje-Gorski Kotar County*	*Croatia*
13.15-13.30	Sead Rešić, Senada Sofić: *Fibonacci numbers and the golden ratio*	*Bosnia and Herzegovina*
13.30-13.45	Damir Horvat, Marija Jakuš: *MayaVi as a tool for presentation of geometric bodies*	*Croatia*
13.45-14.00	Sead Rešić, Olgica Bešić: *Influence of development of computer technologies on teaching in Bosnia and Herzegovina*	*Bosnia and Herzegovina*

14.00-15.00 Lunch Break

Afternoon Session

Chairperson: Ana Mirković Moguš

15.00-15.15	Emil Molnár, István Prok: *Three- and four-dimensional regular 4-solids move in the computer 2- screen*	*Hungary*
15.15-15.30	Ibolya Szilágyiné Szinger: *The role of activity in teaching axial reflection*	*Hungary*
15.30-15.45	Ljerka Jukić Matić, Ivan Matić, Ana Katalenić: *Approaches to learning mathematics in engineering study program*	*Croatia*
15.45-16.00	Željka Milin Šipuš, Maja Planinić, Ana Sušac, Lana Ivanjek: *University students' understanding of graphs in mathematics, physics and other contexts*	*Croatia*
16.00-16.15	Coffee Break	

Chairperson: Ivanka Đeri

16.15-16.30	Aleksandra Čižmešija, Željka Milin Šipuš: *The sine and tangent function - Concept Images of pre-service mathematics teachers*	*Croatia*
16.30-16.45	Ema Kušen, Marija Marinović,: *An approach to Operations Research course in the curriculum for computer science students*	*Croatia*
16.45-17.00	Karmelita Pjanić, Sanela Nesimović: *Fundamental prospective teachers' algebraic knowledge*	*Croatia*

17.00-17.15	Željko Gregorović, Dijana Palinkaš, Margita Pavleković: *Correlation between pupils' managing of graphic data and their level of geometric thinking*	*Croatia*
19.00	Conference Dinner	

SATURDAY, APRIL 20

Morning session

Chairperson: Zdenka Kolar-Begović

8.30-8.45	Plenary lecture István Gubo: *Use of navigation devices in teaching mathematics at primary schools*	*Slovak Republic*
8.45-9.00	Ana Kuzle: *The interrelations of the cognitive and metacognitive factors with the affective factors during problem solving*	*Germany*
9.00-9.15	Marija Juričić Devčić, Velimir Topolovec, IvanMrkonjić: *Future teachers' attitudes toward the use of concept maps*	*Croatia*
9.15-9.30	Ivanka Đeri, Karolina Dobi Barišić, Ljerka Jukić Matić: *The influence of formal education and personal initiatives on the willingness to use ICT in teaching mathematics*	*Croatia*
9.30-9.45	Ivana Đurđević; Ana Mirković Moguš, Ana Katalenić: *Humor in teaching mathematics and computer science courses – yes or no?*	*Croatia*
9.45-10.00	Irena Mišurac Zorica, Maja Cindrić, Marinko Pejić: *Readiness of primary teachers to apply contemporary math teaching*	*Croatia*
10.00-10.30	Coffee Break	

Chairperson: Ljerka Jukić Matić

10.30-10.45	Marija Pisk: *Data processing and presentation in primary school at elementary level - Dealing with teaching experiences*	*Slovenia*
10.45-11.00	Diana Mesić, Sanja Basta: *Open-ended mathematical tasks for primary school youngsters*	*Croatia*

11.00-11.15	Sead Rešić; Zemina Seferović: *Educational effects of the interactive method in styding the Pythagorean theorem in elementary school*	*Bosnia and Herzegovina*
11.15-11.30	Sead Rešić, Nihad Čukle: *Influence of interactive boards in improving teaching of mathematics in high school*	*Bosnia and Herzegovina*
11.30-11.45	Aleksandra Krampač-Grljušić, Ivana Pavleković, Monika Mihaljević: *Touch math*	*Croatia*
11.45-12.00	Edith Debrenti: *The joy of mathematics*	*Romania*

12.00 CLOSING

ABSTRACTS

Regional comparison of the PISA 2009 results in the field of mathematical literacy

Dubravka Glasnović Gracin

Faculty of Teacher Education, University of Zagreb, Croatia

Abstract. This work presents the analysis results on the requirements in PISA 2009 mathematical items. The analysis is based on all 35 mathematical literacy items, which were involved in PISA 2009 assessment. The research objectives were to determine what mathematical contents and activities are required in mathematical PISA 2009 items, to compare Croatian PISA results with the results of other countries in the region (Slovenia, Serbia, Montenegro), and to compare PISA and curricular requirements in Croatia. The results show that PISA 2009 requirements differ from the textbook and curricular requirements in Croatia to a great extent. The paper presents the detailed analysis of requirements in the items, in which the Croatian students showed poor results, compared to the OECD average and other countries from the region.

Keywords: Mathematics education, PISA assessment 2009, mathematical literacy, requirements

Levels of geometric thinking in the second triad of elementary school

Maja Škrbec

Elementary school Notranjski odred Cerknica, Slovenia

Abstract. Dutch mathematician Pierre M. van Hiele developed a theory how to determine a level of geometric thinking of an individual on the basis of solving geometrical problem. He designed and accurately described five levels of geometric thinking. His findings encouraged several researchers to deal with this issue. Van Hiele's findings also affected changes of the curriculum in many countries. We decided to research this problem because in Slovenia no significant research dealing with levels of geometric thinking in elementary school has ever been made. 782 pupils from grades four, five and six of elementary schools solved a test that we had prepared. We found out that the majority (60.7 %) of pupils in the second triad are between level 0 (visual) and level 1 (descriptive) of geometric thinking. Nearly a third (31.7%) of pupils aged 9-11years are on the first level and 4.3 % are on level 0. According to the whole test, only 1.4 % of children were on the second (informal deduction) level of geometric thinking. We found out that pupils know certain properties of geometric figures. But these properties are not logically arranged and moreover, pupils are not able to apply knowledge to new situations. Among other things, we confirmed that there is a difference among classes. Older pupils are at a higher level of geometric thinking than younger ones. We did not notice any difference between genders. We also confirmed the assumption that pupils who have a higher grade in mathematics are at a higher level of geometric thinking. We found out that they are using different levels of thinking, when they are solving problems with different concepts. Using appropriate vocabulary was the biggest problem that the pupils had. They are still confusing basic geometric concepts such as edge and side. This paper briefly presents a model of teaching that aims at improving geometric vocabulary and at the same time promoting higher levels of geometric thinking.

Keywords: geometry, van Hiele, levels of geometric thinking, second triad, encouraging higher levels of geometrical thinking

Didactic model of development of multiplicative concept in mathematics in elementary school

Maja Cindrić[1], Irena Mišurac Zorica[2]

[1]Department for Teacher and Preschool Teacher Education, University of Zadar, Croatia
[2]Faculty of Philosophy, University of Split, Croatia

Abstract. Didactic model of the mathematical concepts based on thoroughly analyzed key points of development, links and development cycles, didactic and methodical approach designed model. Goal is to precisely define all aspects of mathematical concepts, approaches to the realization of these concepts, taking into account the age and cognitive capabilities of the child. Globally didactic model describes the approach to mathematical concepts through two aspects, substantive and procedural. The substantive aspect will list all the key points, concepts, level, learning outcomes, a situation that represents a particular class situation, key issues, representation, possible misconceptions, the age at which it expected to be, a description of the cycle and connectivity, while the procedural aspect based on description facilities through which it will develop competence in the spectrum of mathematical processes. Example of didactic model of development of mathematical concepts in this paper will describe the multiplicative concept through lower elementary grades, and the analysis of how the textbooks used in Croatian schools presents multiplicative concept comparing to the described model.

Keywords: mathematical concepts, multiplicative structures

Use of navigation devices in teaching mathematics at primary schools

Gubo István

Faculty of Economics, Department of Mathematics and Informatics, J. Selye University, Komarno, Slovak Republic

Abstract. GPS (Global Positioning System) is one of the modern technologies which have been expanding in our everyday life. Not only can the navigation devices be used in industry, agriculture or during outdoor free-time activities, but also in teaching certain subjects at school.

The purpose of this article is to outline the possibilities of using navigation devices in teaching mathematics. We introduce an activity which was carried out at primary schools in Hungary. During this activity, the pupils have learned how to use a hand-held GPS receiver, and then solved measuring tasks around the school using this receiver.

Keywords: teaching mathematics, GPS, navigation device

The role of activity in teaching axial reflection

Szilágyiné Szinger Ibolya

Eötvös József Főiskola, Baja, Hungary

Abstract. The basis of learning geometric transformations, thus axial reflection as well, is inductive cognition based on gaining empirical knowledge. Starting out from the concrete and gathering experience from various activities, such as folding, clipping, drawing, and the use of mirror will finally lead to the formulation of general relationships.

In the junior classes pupils observe the reflected image of different geometric figures in space and plane by using plane mirror by gaining experience in a playful way. They construct reflections of geometric solids and they produce the reflections of simple plane figures and axial reflections my means of folding, clipping and drawing.

Activities have an equally important role in teaching axial reflection in junior high school as well. Producing axially reflected shapes is possible in several ways: by means of moving, using transparent paper and drawing on grid and by construction.

Keywords: teaching mathematics, axial reflections, activity

Formal language of mathematics and logic in elementary school instruction of mathematics

Miljenko Stanić

Faculty of Teacher Education, University of Rijeka, Croatia

Abstract. This paper presents a didactic tool for introducing pupils of early school age to the formal language of mathematics, logic and science. The tool is a virtual world on the computer screen. The picture – world consists of a chessboard on which different figures are laid. Properties are attached to every indivudual – figure and their positions on the chessboard are favourable for establishing various relations. The idea is based on a computer aid, *Tarski' s World,* designed for the instruction of mathematical logic for philosophy students on American universities. Our world, named *Our Little Village* (the name has been borrowed from the Czech movie of the same title, directed by J. Menzel) is coloured, fun and suitable for creating children' stories, from which a trained teacher can abstract a lot of mathematical and logical principles/validities expressed by mathematical –logical formulas. In the same way the world can be approached reversely, so that, based on a list of formulas- statements – principles, a new configuration is created on the chessboard. This aid is a good tool for the introduction of pupils to mathematical structures such as the naive set theory and mathematical logic and is a good means of solving logical equations or riddles from recreational mathematics.

Keywords: Tarski's World, the naive set theory, mathematical logic, formalisation, mathematical structures

Center for mathematics in living room of the pre-school education institution

Višnja Vekić Kljaić

Pre-school education center Osijek, Kindergarden Pčelica, Croatia

Abstract. Pre-school education aiming at development encompasses design of the Center for Mathematics and manipulative games in the living room for children. Materials of these centers support child exploration, imagination and manipulation i.e. these are the means that help children to learn how to compare, coordinate, calculate and categorize. Their value from the aspect of child's development are that the activities in this center help children develop their intellectual abilities, fine motorics as well as their eye and hand coordination. Through negotiating and problem solving they learn social skills as well.

Pre-school children reveal mathematic notions through rhythm and repetitive movement, colour, sound etc. Ready made didactic materials can be used for this purpose but also the means that kindergarden teachers make themselves are equally efficient. The main feature of the materials are that they offer children concrete, systematic experience in counting, enumeration and comparing.

In Centers organized in this way the task of the teacher is to equip the living room with different materials that offer a number of possibilities for developing skills of mathematical reasoning, observe children playing and use every opportunity to explain the notions in order to help children arrive at mathematical conclusion on their own.

The aim of the paper is to get insight into the organisation and equipment of centers for mathematics and pre-school teachers' idea of their role in center equipment.

Keywords: centers for mathematics, developmentally appropriate practice, pre-school child, pre-school teachers

The joy of Mathematics

Debrenti Edith

Partium Christian University Oradea, Romania

Abstract. According to George Pólya once someone has discovered the joy of *mathematics*, they will never forget it.

In this research paper I present a set of activities that were carried out during the previous academic year among second grade student (aged 7- 8) living in Oradea. The purpose of these activities is to make the student enjoy and like studying mathematics and to make them motivated in gaining mathematical knowledge. Moreover I was applying methods that can make math classes interesting and efficient.

Different interactive methods were applied using elements of Drama Pedagogy, didactical games and some components of the cooperative method. Our main standpoint was to let student be a part of the process of solving the mathematical problem and to encourage them to cooperate as often as possible. Our intention was to show student that they should not be afraid of trying to solve the problems and they should not be afraid of failure. Thus they can enjoy solving problems even on their own.

Keywords: experience, skill development, action-oriented and child-centered learning, sense of achievement, intrinsic motivation

Application of the elements of Vedic mathematics in classes with Roma pupils

Goran Lapat[1], Lidija Eret[2]

[1]Faculty of Teacher Education, University of Zagreb, Croatia
[2] Eugen Kvaternik Elementary School, Velika Gorica, Croatia

Abstract. Work with Roma children raises special challenges for educators and teachers who during their pre-service training have not been prepared for the specific work within the culture of the Roma community and the Roma language and in particular for the flexibility in work with Roma pupils. Interest of Roma pupils in educational process is a challenge to which we have tried to respond by introducing elements of Vedic mathematics in tuition. This article provides a methodological proposal which shows a connection of an active approach of Roma students' mathematical learning with the introduction of the Vedic way of calculating in additional mathematics classes and extracurricular activities.

Keywords: Vedic mathematics, Roma, additional mathematics classes, extracurricular activities

Influence of development of computer technologies on teaching in Bosnia and Herzegovina

Sead Rešić[1] and Olgica Bešić[2]

[1]Faculty of Science, Department of Mathematics, University of Tuzla, Bosnia and Herzegovina
[2]Elementary school Bukinje Tuzla, Bosnia and Herzegovina

Abstract. Our times are characterized by strong changes in technology that have become reality in many areas of society. When compared to production, transport, services, etc education, as a rule, slowly opens to new technologies. However, children at their homes and outside the schools live in a technologically rich environment, and they expect the change in education in accordance with the imperatives of the education for the twenty-first century. In this sense, systems for automated data processing, multimedia systems, then distance learning, virtual schools and other technologies are being introduced into education. They lead to an increase in students' activities, quality evaluation of their knowledge and finally to their progress, all in accordance with individual abilities and knowledge. In this thesis we try to present the advantages and disadvantages of integrating ICT in teaching mathematics, covering the following topics: the need of introducing and the goals of innovative teaching, planning integration of computers in the teaching process -integration models, resistance to integration, ways of applying ICT in education and teaching mathematics (through the classes of acquiring the new teaching material, evaluation classes and testing and evaluation of students' knowledge as well). In order to prove the need of introducing ICT in the teaching process, the survey of students and teachers on the current representation of ICT in teaching was conducted. Furthermore, some mathematics lessons were held supported by the use of ICT and then the research was conducted about computer application in mathematics teaching.

Keywords: education, multimedia systems, distance learning, virtual schools, mathematics and computers, lectures, exercises, integration, survey, research

On the usage of interactive whiteboards in the teaching of mathematics in secondary schools in Primorje-Gorski Kotar County

Vedrana Mikulić Crnković and Sanja Rukavina

Department of Mathematics, University of Rijeka, Croatia

Abstract. In the year of 2005 Ministry of Science, Education and Sports of the Republic of Croatia has launched the project procurement of interactive whiteboards for elementary and secondary schools. Accordingly, from 2006/2007 secondary schools have been systematically equipped with "smart boards" and introduced with on-line interactive educational contents. Moreover, the National Educational Standard specifies interactive whiteboard as the required equipment at the school level. We can find numerous statements of teachers who are enthusiastic about the introduction of interactive whiteboards in the classroom, but there are also those who doubt the positive impact of this type of innovation.

In order to determine to what extent is the usage use of interactive whiteboards widespread in teaching of mathematics in PGC secondary schools, we conducted a research among secondary school teachers of mathematics in PGC. Furthermore, the aim was to examine the ways in which the interactive whiteboards are used in teaching of mathematics and what are the possible reasons for not using them. In addition, we have examined how many of undergraduate students of Mathematics in Department of Mathematics, University of Rijeka, have met with the usage of interactive whiteboards in math classes in secondary school. Given the fact that most of them are prospective teachers of mathematics, we were interested in their attitudes, based on their personal experience, towards the usage of interactive whiteboards in teaching of mathematics.

Keywords: teaching of mathematics, interactive whiteboard

Influence of interactive boards in improving teaching of mathematics in high school

Sead Rešić[1] and Nihad Čukle[2]

[1]Faculty of Science, Department of Mathematics, University of Tuzla, Bosnia and Herzegovina
[2]High School of Economic Travnik, Bosnia and Herzegovina

Abstract. Interactive (smart) whiteboards are an ideal solution for those who want to increase the quality of teaching, to encourage and motivate the students and help to achieve the desired results. The features and benefits of interactive whiteboards are very large. It can be used as a regular whiteboard, or connect to a computer via a serial cable, USB or wireless, and take full advantage of its interactivity.

With each interactive whiteboard comes accompanying software that includes a wide variety of contents, including content related to teaching and learning in mathematics. With the help of interactive whiteboards and expertise teaching math capabilities are unlimited. The limitation of interactive whiteboards utility is just our imagination. Interactive whiteboard attracts and holds the attention of students creating compelling of interactive lectures where everyone loves to participate.

We are interested in how smart whiteboard functions in an educational environment (technical description), and how the introduction of such a smart whiteboard in the classroom reflects the subjects of mathematics and teaching methodology (methodological description).

Students through the questionnaire express their impressions about this way of learning, or whether the use of interactive content in the teaching of mathematics contributed it to be their favorite subject. Conducted research suggests that before utility of interactive whiteboards students have not expressed sympathy for the subject of mathematics, and the most of them thought that it was a difficult subject. After installing and applying the same, the results of the final questionnaires show

the changes in the positive direction in terms of motivation and learning approach in mathematics, and respectively:

- students demonstrate themselves more independent when acquiring new teaching
- students independently reach conclusions
- slmost all of the students are active in the class
- students express less need for additional explanations from the teacher in front of the whiteboard
- students show more interest and motivation for learning mathematics
- students with their own initiative visit websites with digital materials at home
- students are more satisfied with the mathematics teaching (as they are using the computer more often)
- students can ask each other's help too, and they help each other more

The results of the final interview (the first grade of high school, 150 students, 2012.):

Students listed their impressions about this way of learning, whether the use of interactive whiteboards in the teaching of mathematics contributed it to be their favorite subject, etc. The first results (the traditional approach) show that 65% of the students, when they encounter a problem in solving the tasks, address to his/her teacher, and now it is 35%; also 18% tried to find a solution without the help of teachers or parents earlier, and is down to 65% now. From this it follows that provided interactive content increased motivation for individual work and finding solutions without additional help.

The results of the interesting facts of mathematics teaching in the first survey (the traditional approach) were really against existing teaching mathematics, or over 75% of students said they do not like mathematics, and over 80% believed that mathematics is a difficult subject, while nearly 90% thought that the teaching of informatics much more interesting than teaching mathematics. Results presented later modern methods in teaching mathematics are really much better. Even 84% believed that the teaching of mathematics with corresponding interactive contents

can be just as interesting as the teaching of informatics, and that over 76% of students said they prefer this way of learning in addition to traditional education.

Eventually, the students were given evaluation for the classes supported by interactive content. Only 8% of the respondents consider and state that it is not satisfied with the aspect of teaching, and 12% of those surveyed students are assessed with grade 2, and 20% with grade 3. With the grade 4 were 26% of respondents and with grade 5 were answered by the most of respondents 34%.

With this issue the science is not sufficiently dealt with in BiH, and what has already been written about it is incidentally and not enough interpreted.

Keywords: interactivity, discovery, GeoGebra, virtuality, constructivism, *multimedia*

MayaVi as a tool for presentation of geometric bodies

Damir Horvat and Marija Jakuš

Faculty of Organization and Informatics, University of Zagreb, Croatia

Abstract. It is a common occurrence to talk about the necessity of implementing IC technology within math learning process. Today, learners' generations are dealing with technology from early childhood so, it is essential to adapt teaching methods to their needs and habits.

In this paper, we shall introduce MayaVi - software for animation, modelling and simulation. Through various examples, we will provide just one small segment of the MayaVi usage - the presentation of geometric bodies and intersection of geometric bodies with plane. This area of mathematics is a part of the national curriculum for the 2nd grade of secondary (technical) schools. As in teaching, the body sketches are mainly drawn by hand, we believe that an alternative, more dynamic and interactive approach could refresh and enrich the math teaching process.

Keywords: math, geometric bodies, MayaVi

Reflected actionability – acquisition of reflection competences for prospective teachers

Markus Alexander Helmerich

Didaktik der Mathematik, University of Siegen, Germany

Abstract. An important aim of teachers' education at University of Siegen is to strengthen prospective students in their ability to stay capable of action in uncertainty and conflicting situations in teaching and learning processes at school as well as in their own mathematical education. This could be achieved by building up the reflection competence of students in respect to their image of mathematics, their beliefs on what is important in mathematics and didactics of mathematics and how to teach and how to learn mathematics. From surveys among first year students and interviews with students in mentoring situations it was possible to retrieve fundamental areas of tension such as "form and content", "rigour and intuition" and "process and product", which often leads to the challenge to make the "right" decision in such conflicting areas.

These dialectical concepts of tension are used as orientation marks in lectures and initial points for reflecting processes. Furthermore I will give insight in a case study, showing examples of reflecting essays. Accompanying a university class, students were confronted with statements or questions on mathematics and teaching. They were then asked to write short reflecting essays on their beliefs and opinions on these topics. It is important for students to position themselves in the wide range of possible actions in teaching and learning processes. To know the own position and belief is a crucial basis to achieve skills in decision-making and responsibility for and in teaching and learning situations in mathematics. Furthermore this gives the background to sequentially enhance the ability of reflecting learning and teaching processes and to develop a relaxed attitude in pressure situations in classroom action.

Keywords: reflection competence, teacher education, reflected actionability

Dependence of the problem solving strategies on the problem context

Vida Manfreda Kolar and Tatjana Hodnik Čadež

Faculty of Education, University of Ljubljana, Slovenia

Abstract. Of all mathematical processes generalization is considered one of the most important ones. We need to distinguish between two aspects of generalization: seeing the general in the particular, or seeing the particular in the general. In the first case we are speaking of inductive reasoning, whereby the observation of special cases leads one to suspect very strongly that some general principle is true. Deductive reasoning, on the other hand, is a process of inferring conclusions from the known information (premises) based on formal logic rules. For the purpose of our research we would like to distinguish between the following types of generalization: generalization through induction, generalization through perceiving recurrence and generalization through generalizing the reasoning.

In the paper the results of the study on primary teacher students' competences in generalization are presented. Our research was focused on delving into the students' problem solving strategies in relation to the problem type. We compared students' problem solving strategies when solving two different types of problems: a conceptual one and a procedural one in relation to two types of generalisation, i.e. the inductive reasoning and the generalization through generalizing the reasoning. The research results lead to some significant conclusions. We conclude that the type of generalization is related to the type of the problem: if a relationship between the problem and the mathematical concept can be established, the reasoning type of generalization prevails. If, on the other hand, it is not possible to establish clear connection between the problem and a certain mathematical concept, working with mere numbers prevails, and, consequently, the inductive type of generalization.

Keywords: problem solving; procedural, conceptual problems; problem solving strategy; generalisation; inductive reasoning; generalisation through generalizing the reasoning

Three- and four- dimensional regular 4-solids move in the computer 2-screen

Emil Molnár and István Prok

Department of Geometry, Institute of Mathematics, Budapest University of Technology and Economics, Budapest, Hungary

Abstract. In previous works (see [1], [2], [3]) the authors extended the method of central projection to higher dimensions, namely, for $\mathbf{E}^4 \to \mathbf{E}^2$ projection from a one dimensional centre figure, together with a natural visibility algorithm. All these are presented in the linear algebraic machinery of real projective sphere $\mathcal{PS}^4$ or space $\mathcal{P}^4(\mathbf{V}^5, \boldsymbol{V}\ 5, \sim)$ over a real vector space $\mathbf{V}^5$ for points and its dual $\boldsymbol{V}_5$ for hyperplanes up to the usual equivalence $\sim$ (expressed by multiplication by positive real numbers or non-zeros, respectively).

In this presentation we further develop the exterior (Grassmann) algebra method (with scalar product) by computer to other effects of illumination. For example, regular 4-polytopes move in the computer 2-screen with visibility and shading of 2-faces, on the base of the homepage http: //www.math.bme.hu/~prok

Our illustrations below are made on the base of the above homepage, where we further develop this attractive topic

This work is dedicated to the Memory of Erno Molnár, the Father and first Mathematics Teacher of the presenter, on His 100th Birthday.

Keywords: animation of regular figures for aesthetics of mathematics

References

[1] Katona, J., Molnar, E. (2009), *Visibility of the higher-dimensional central projection into the projective sphere*, Acta Math. Hungar. 123(3), 291–309.

[2] Katona, J., Molnar, E., Prok I. (2008), *Visibility of the 4-dimensional regular solids, moving on the computer screen*, In Proc. 13th ICGG (Dresden, Germany).

[3] Katona, J., Molnar, E., Prok I., Szirmai, J. (2011), *Higher-dimensional central projection into 2-plane with visibility and applications*, Kragujevac Journal of Mathematics, 35(2), 249–263.

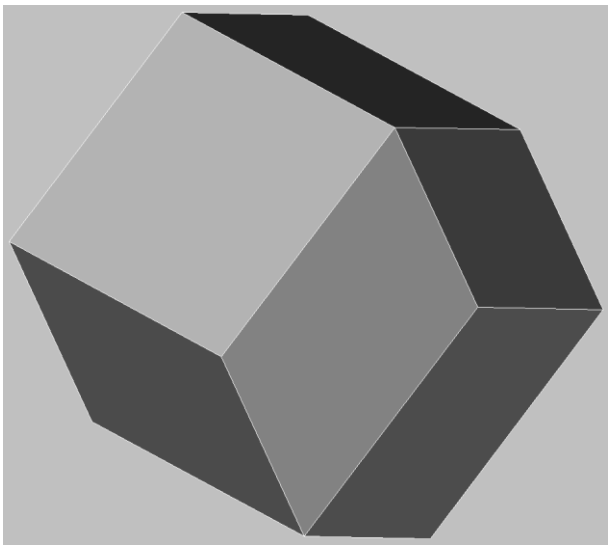


Figure 1. The 4-cube with Coxeter-Schläfli symbol (4, 3, 3)

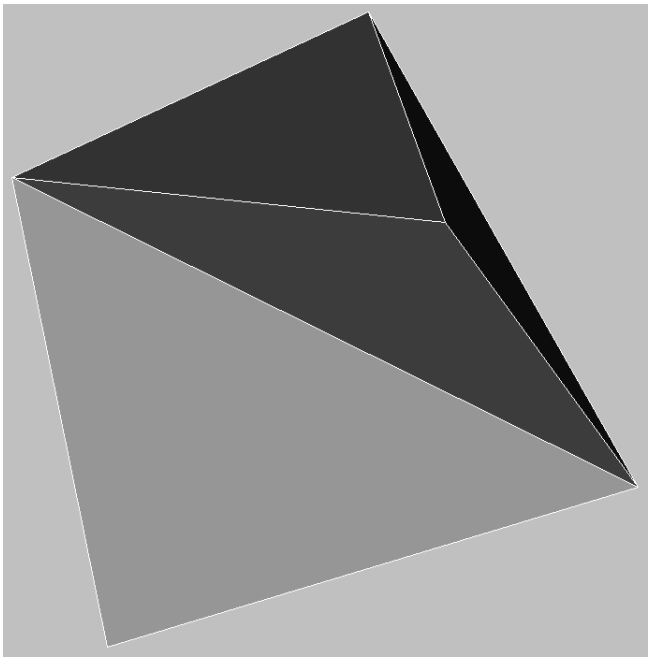


Figure 2. The 4-Simplex with Coxeter-Schläfli symbol (3, 3, 3)

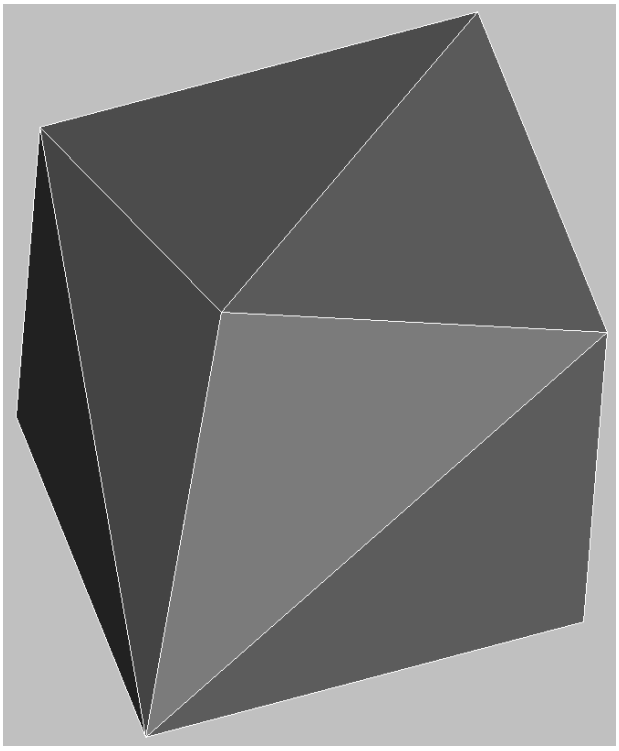


Figure 3. The Cross-polytope with Coxeter-Schläfly symbol (3, 3, 4)

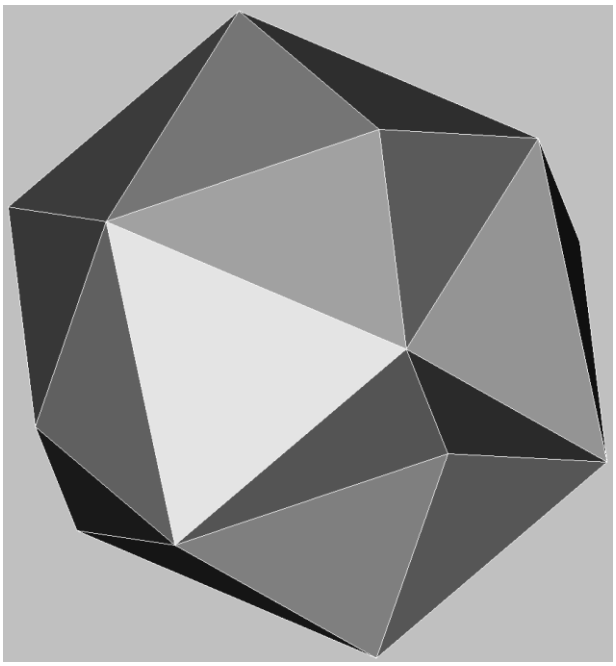


Figure 4. The 24-cell with Coxeter-Schläfli symbol (3, 4, 3)

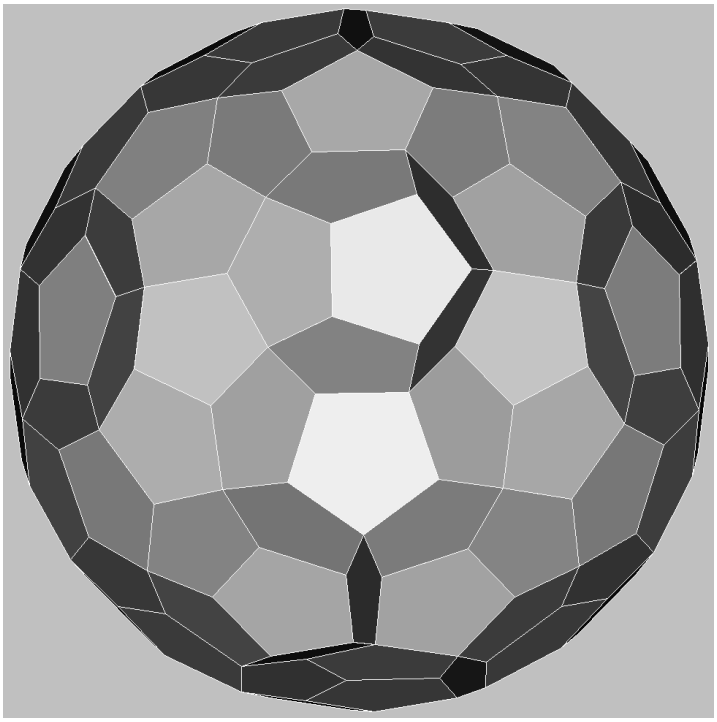


Figure 5. The 120-cell with Coxeter-Schläfli symbol (5, 3, 3)

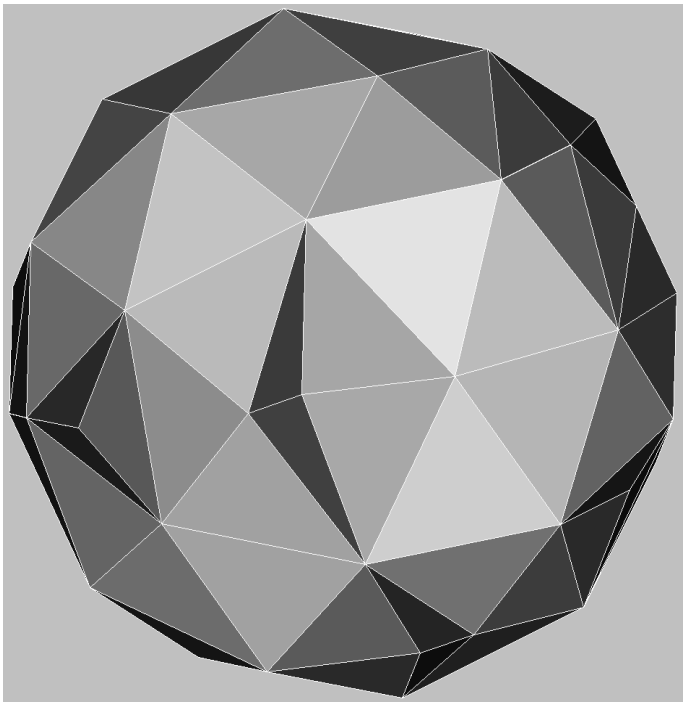


Figure 6. The 600-cell with Coxeter-Schläfli symbol (3, 3, 5)

Approaches to learning mathematics in engineering study program

Ljerka Jukić Matić[1], Ivan Matić[2] and Ana Katalenić[3]

[1,2]Department of Mathematics, Josip Juraj Strossmayer University of Osijek, Croatia
[3]Faculty of Teacher Education, Josip Juraj Strossmayer University of Osijek, Croatia

Abstract. The students' approaches to learning have been significantly researched in the last few decades, particularly since Marton and Säljö in 1979. elaborated about a deep/surface approach dichotomy. The above dichotomy appears to be very useful in the assessing of teaching, based on which one can obtain parameters according to which teaching can be improved. Precisely for these purposes the term approach to learning is more suitable than the term learning style. Learning style addresses ability-like dimensions while the term approach means that person can choose to learn in different ways depending on his/her motivation, the nature of the course taken and subject-matter, as well as the host of other variables. We use The Approaches and Study Skills Inventory for Students (ASSIST), developed by N.J. Entwistle, to investigate learning approaches of the first-year undergraduate engineering students in compulsory mathematics course. We emphasize that this inventory considers an additional type of approach, the strategic one. Inventory explored which of three approaches to learning (deep, strategic or surface) was chosen the most to cope with demands of the specific mathematics course, and how the chosen approach relates to the students' grades obtained in the course. The results showed that majority choose strategic approach, what might indicate that it was the nature of the mathematics course that resulted in this approach. Also, in this paper we discuss the potential main factors that could result in such a selection.

Keywords: approaches to learning, deep, surface, strategic

University students' understanding of graphs in mathematics, physics and other contexts

Željka Milin Šipuš[1], Maja Planinić[2], Ana Sušac[3] and Lana Ivanjek[4]

[1]Department of Mathematics, Faculty of Science, University of Zagreb, Croatia
[2,3,4] Department of Physics, Faculty of Science, University of Zagreb, Croatia

Abstract. Scientific data are very often presented through graphs which allow the skilled user to quickly extract information on the data presented. In physics especially, the analysis of data, such as dependence of variables appearing in interpretation of physical phenomena presented by a graph, relies heavily on interpretation of graph slopes and areas under the graphs. Students meet these concepts as purely mathematical concept in mathematics education but also within their application in different contexts and disciplines (e.g. physics). The physics curriculum often expects that understanding of these concepts is well developed in mathematical courses, whereas physics teachers report on students' lack of mathematical knowledge and the ability to apply their knowledge in physics ([3]). This is the problem of a transfer of learning which is concerned with the student's ability to apply what has been learned in one context to a new context. It is one of the biggest challenges in education ([2]), especially valid for mathematical knowledge which is considered as a prerequisite for science education. It is not only the problem of recognizing mathematical concepts in different contexts, but also of recognizing problems common to mathematics and other disciplines and problems specific to the specific discipline. In physics, for example, problems stripped off their physical context „often involve representations that deviate from those typically used in the mathematics domain"(„physicsless physics questions", [1]).

In the study presented, our aim was to investigate whether and how students recognize mathematics to be used in contexts problems which involve analysis of graph slopes and areas under the graphs. We summarize our findings on understanding of these concepts across three different domains, mathematics, physics (kinematics) and context other than physics (economy, biology, everyday

life). The last domain did not require any additional context-dependent knowledge. We have collected data on 385 first year students of mathematics and physics on the test developed by authors consisting of eight set of parallel questions. Questions were parallel in the sense that the (final) required mathematical procedure was the same in each set of three items. However, whereas the problem of areas under the graphs in mathematics domain was concerned with numeric calculation of an area, for the context other than physics it involved the interpretation of an area under the graph as a cumulative growth of a quantity. Such an interpretation lies in the fundamentals of concept of integration ([4], [5]). The analysis of results of our study suggests that the concept of a graph slope is equally difficult in all three areas, whereas the difficulty of the concept of an area under graph differs across domains. Furthermore, the findings suggest that students' mathematical knowledge is not the only important factor for student's success in solving graph problems in physics or other contexts.

Keywords: graphs, mathematics, physics, slope, area

References

[1] Christensen, W.M., Thompson, J.R. (2012), *Investigating graphical representation of slope and derivatives without a physics context,* Physical Review Special Topics 8 (2), 023101.

[2] Michelsen, C. (2005), *Expanding the domain – Variables and functions in an interdisciplinary context between mathematics and physics*, In Beckmann, A., Michelsen, C., Sriraman, B. (Eds.), Proceedings of the 1st International Symposium of Mathematics and its Connections to the Arts and Sciences, Schwäbisch Gmünd, Germany, 201-214.

[3] Planinić, M., Milin Šipuš, Ž, Katić, H., Ivanjek, L., Sušac, A. (2012), *Comparison of student understanding of line graph slope in physics and mathematics*, International Journal of Science and Mathematics Education, 10 (6), 1393-1414.

[4] Tall, D. (1997), *Functions and Calculus*, In A. J. Bishop et al. (Eds.), International Handbook of Mathematics Education, 289-325.

[5] Thompson, P. W., Silverman, J. (2007), *The Concept of Accumulation in Calculus*, In M. Carlson, C. Rasmussen (Eds.), Making the connection: Research and teaching in undergraduate mathematics, 117-131.

The sine and tangent function - concept images of pre-service mathematics teachers

Aleksandra Čižmešija and Željka Milin Šipuš

Department of Mathematics, Faculty of Science, University of Zagreb, Croatia

Abstract. According to the actual Croatian high school mathematics curriculum, trigonometric functions of a real argument are introduced and taught in the 3rd grade of grammar and of four-year technical high schools and assessed at the national level at the higher level of State Matura after the completion of fourth grade. Results show that students' performance on tasks involving this mathematical content is not satisfactory although a significant number of lesson periods has been allocated for its comprehension. Considering that teacher quality is an important school-based factor affecting students' achievements, we have conducted a survey investigating the basic concepts and procedures related to trigonometric sine and tangent functions among the university mathematics education students in Croatia. We have acquired data over entire populations of the 3rd year and of the 5th year students of mathematics education programmes at the largest Croatian university on a questionnaire with open-ended questions and on related semi-structure interviews. Pre-service teachers' responses were then analyzed against the theoretical framework of concept image vs. concept definition (Tall & Vinner, 1981). The obtained results suggest that the right triangle trigonometry and degree measure make significant part of their concept images of sine function, whereas the tangent function is seen dominantly only algebraically, that is, as a ratio of sine and cosine, without referring to its geometric interpretation in circle trigonometry. This finding is most evidently seen in pre-service teachers' approaches to solving simple trigonometrice quations and inequalities, such as $\sin x = -0.5$, $\tan x > -1$ or $\sin x < \cos x$. Moreover, the evidence gained shows that such tasks are prevalently solved by employing procedural knowledge rather than conceptual. Some of these findings on Croatian data set confirm and further extend earlier results in Fi, 2006, and Topçu, Akkoç, Yılmaz & Önder, 2006.

Keywords: Mathematics education, trigonometric functions, concept image

References

[1] Fi, C. (2006), *Preservice secondary school mathematics teachers' knowledge of trigonometry: Cofunctions*, In Alatorre, S., Cortina, J.L., Sáiz, M. & Méndez, A. (Eds.), Proceedings of the 28th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Mérida, México: Universidad Pedagógica Nacional, Vol. 2-833.

[2] Tall, D. & Vinner, S. (1981), *Concept image and concept definition in mathematics with particular reference to limits and continuity*, Educational Studies in Mathematics, 12, 151-169.

[3] Topçu T., Kertil M., Akkoç H., Yılmaz K. & Önder O. (2006), *Pre-service and in-service mathematics teachers' concept images of radian*, In Novotná, J., Moraová, H., Krátká, M. & Stehlíková, N. (Eds.), Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education, Vol. 5, 281-288.

An approach to Operations Research course in the curriculum for computer science students

Ema Kušen and Marija Marinović

Department of Informatics, University of Rijeka, Croatia

Abstract. Operations Research is a compulsory course taught in the winter semester at the Department of Informatics, University of Rijeka. The profile of students that are enrolled into this course varies. It includes single major computer science students (a teacher training module, a module Information and Communication Systems and a module Business Informatics), and double major students of computer science in combination with History, English Language, German Language and other combinations. The problem which we often face with is different level of pre-knowledge of Mathematics for different groups of students. Based on the results of the past surveies and the student's attitude, both single major and double major students of Informatics lack in motivation to study Mathematics courses which are offered at undergraduate and graduate levels of studies. These are the courses in which students get lower marks.

According to the research results presented in the literature, it has been concluded that there are factors on which professors can influence in order to improve the students' performance in the course. One of such is a level of complexity of the presented materials. Guided by the fact that holding lectures in teaching Mathematics courses is not sufficient for students to gain knowledge, we decided to develop a set of activities by which we tried to overcome the lack of pre-knowledge and explain complex and abstract concepts which are the basis of Operations Research in a more approachable manner. While teaching this course we used the ICT which is common to the computer science students.

In this article we will present our approach of designing and teaching the course Operations Research with the help of the ICT and compare results of a student pass rate for the previous three academic years.

Keywords: ICT, computer science students, Operations Research, instruction design, online activities

Fundamental prospective teachers' algebraic knowledge

Karmelita Pjanić[1] and Sanela Nesimović[2]

[1]Faculty of Educational Studies, University of Bihać, Bosnia and Herzegovina
[2]Faculty of Educational Sciences Sarajevo, University of Sarajevo, Bosnia and Herzegovina

Abstract. There is no question that elementary school teachers are the ones who lay the foundations of mathematical knowledge. The choice of future career of their students is largely dependent of the strength of given mathematical foundations. But, what is the quality of the mathematical foundations of prospective elementary school teachers? Are the fundamental mathematical knowledge firm and deep enough to allow prospective elementary school teachers to practice methodical processing of these knowledge?

In order to provide insight on these questions, the research on basic algebraic skills of prospective elementary school teachers was conducted. Sixty 3rd year students at Pedagogical faculty in Sarajevo participated in the research. Special attention was given to diagnose difficulties that prospective elementary school teachers show in the case of different representations of basic algebraic concepts and their transformations. For this purpose, prospective elementary school teachers solved the tasks of recognition, representation, transformation and interpretation of basic arithmetic and algebraic expressions and properties.

Qualitative analysis of the data indicates the difficulties and misconceptions rooted in primary education, and points to the necessity of carrying out modifications in the content of pre-service education of prospective teachers.

Keywords: algebraic knowledge for teaching, concept representation, concept transformation

Correlation between pupils' managing of graphic data and their level of geometric thinking

Željko Gregorović, Dijana Palinkaš and Margita Pavleković

Faculty of Teacher Education, Josip Juraj Strossmayer University of Osijek, Croatia

Abstract. Since the year 2010 the National Curriculum Framework[1] in Croatia has, among other, defined the general goals for the educational area of mathematics, as well as the expected achievements of pupils at the end of each educational cycle with regards to two segments: mathematical processes and mathematical concepts. In primary school the first educational cycle is completed at the age of ten. Accordingly, in the aforementioned document it is anticipated that at the end of the first cycle within the mathematical concept labelled *Data* ten-year-old pupils will be able to:

- *collect, classify, and organise data culled from their everyday lives, and present them using simple tables, pictograms (i.e. graphic charts), and bar charts,*
- *read and interpret data presented in simple tables , pictograms and bar charts, ...*

Due to the fact that the mathematics curriculum in Croatia does not anticipate the development of the aforementioned knowledge and skills until the seventh grade of primary school, the students of the fifth year of teacher studies attempted to investigate:

1. how to encourage lower primary school pupils to successfully manage graphic data,
2. whether the level of geometric reasoning of ten-year-olds is correlated to their data reading skills, as well as the skills of interpreting and presenting data.

[1] National Curriculum for Preschool Education, General Compulsory and Secondary School Education, http://public.mzos.hr/Default.spx?sec=2685(1.3.2012.)

We sought out these answers in collaboration with the students by working with fourth grade pupils within a pilot research.

Keywords: levels of geometric thinking/reasoning according to van Hiele, graphic presentation of data, reading and interpreting graphic data, student competences

Future teachers' attitudes toward the use of concept maps

Marija Juričić Devčić[1], Velimir Topolovec [2] and Ivan Mrkonjić[3]

[1,3] Faculty of Teacher Education, University of Zagreb, Croatia
[2] Department of Informatics, University of Rijeka, Croatia

Abstract. This article discusses the experiences with the use of concept maps in teaching mathematics in the first semester at the Faculty of Teacher Education of the University of Zagreb. We present and analyze the results of a survey conducted among students that reflects their attitudes toward the use of concept maps. The results are compared with the results of previously conducted survey among future teachers in the United Arab Emirates. Finally, we give directions for our future work and research.

Keywords: concept maps, mathematics teaching, teacher education, students' attitudes

The interrelations of the cognitive and metacognitive factors with the affective factors during problem solving

Ana Kuzle

Faculty of Computer Science, Electrical Engineering and Mathematics, Department of Mathematics, University of Paderborn, Germany

Abstract. Problem solving is an extremely complex human endeavor that involves a complex interplay between cognition and metacognition. Students' rich store of mathematical knowledge and facts, ability to access and organize knowledge already possessed, plan strategies for implementing what is known, and monitor the effectiveness of these strategies are one of many factors adversely affecting problem-solving performance. Nevertheless, affective attributes, such as motivation, interest, pleasure, impatience, anxiety, beliefs, and persistence in problem solving are of high significance and may influence students' problem-solving performance (Goldin, 2000). Moreover, effective management of negative affective behaviors, such as anxiety and frustration, is instrumental for participants' perseverance during problem solving (Carlson & Bloom, 2005). Veenman, Van Hout-Woulters, and Afflerbach (2006) contend that we as researchers need to focus on understanding how individual differences and contextual factors interact with learning, and problem-solving processes.

With these considerations in mind, in this paper I focused on identifying affective behaviors, and describing situations where a particular affective behavior occurred in the context of problem solving with technology. Moreover, special attention was given to elucidate mechanisms participants used to cope with both productive, and counterproductive affective behaviors, and its effect on subsequent problem-solving endeavors. The results of the study showed that effectiveness of solution approaches was dependent on the presence of managerial decisions. Cognitive problem-solving actions not accompanied by appropriate metacognitive monitoring actions of affective behaviors appeared to lead to unproductive efforts. Redirection and reorganizing of thinking in productive and positive directions

occurred when metacognitive actions guided the thinking and when affective behaviors were controlled.

Keywords: affective behaviors, mathematical behavior, metacognition, problem solving, regulation

The influence of the formal education and personal initiatives for the willingness to use ICT in teaching mathematics

Ivanka Đeri[1], Karolina Dobi Barišić[2] and Ljerka Jukić Matić[3]

[1,2] Faculty of Teacher Education, Josip Juraj Strossmayer University of Osijek, Croatia
[3]Department of Mathematics, Josip Juraj Strossmayer University of Osijek, Croatia

Abstract. Application of computer technology in the classroom is a challenge for every teacher who wants to follow the development of the information society. Teacher's competence to work with computer technology certainly depends on formal education but also on personal initiatives.

This paper presents the results of study on the willingness of mathematics teachers to use ICT in teaching mathematics in the different educational cycles with respect to their competences. Teachers are aware of the need to use ICT, but point out the shortcomings of their formal education, that shows progress, and emphasize a great need for personal initiatives and further education.

Keywords: mathematics, ICT, IT competences, formal education, personal initiative

Humour in teaching mathematics and computer science courses – yes or no?

Ivana Đurđević, Ana Mirković Moguš and Ana Katalenić

Faculty of Teacher Education, Josip Juraj Strossmayer University of Osijek, Croatia

Abstract. The aim of this study was to examine the role of humor in teaching mathematics and computer science courses at Faculty of Teacher Education in Osijek. The paper explores whether the humor is used and in which intensity it is used as an effective mean of teaching and communication at mathematics and computer science classes, and it also identifies which acceptable form of humor is most often used. Furthermore, the paper investigates whether the use of humor in teaching computer science and mathematics courses has a stimulating effect on the environment for learning in the sense of creating a nice "classroom" climate without stress. During this research two questionnaires were created and used. One of the designed questionnaires was used to test students' attitude about the use of humor in teaching. In fact we wanted to see whether or not students participated in the teaching process which included humor in some of its acceptable forms, was the effect of that humor stimulating for their learning and did it have impact on overall impression of the course where the humor was used and what effect did the use of humor had on communication with the teacher who has using it. The second questionnaire was used to collect data from teachers themselves, and it was based analogous to the first questionnaire. Answers obtained by the means of used questionnaires were analyzed and the results showed that although teachers of mathematics and computer science courses at Faculty of Teacher Education in Osijek recognize the benefits of using humor in the classroom, they use it rarely and occasionally in the practice.

Keywords: humor in the classroom, improving the learning environment, attitude of students, the impact of humor in communication

Readiness of primary teachers to apply contemporary math teaching

Irena Mišurac Zorica[1], Maja Cindrić[2] and Marinko Pejić[3]

[1] Faculty of Philosophy, University of Split, Croatia
[2] Department for Teacher and Preschool Teacher Education, University of Zadar, Croatia
[3] Faculty of Teacher Education, University of Sarajevo, Bosnia and Herzegovina

Abstract. Observing pupils results in mathematics in Bosnia and Herzegovina and Croatia we are confronted with bad grades, much of negative attitude of society towards math and poor results that pupils obtained in the various external evaluation. Poor results require reconsideration of all the parameters that have an impact on the results of mathematical education, and one of the most important is a teacher. With modes, enthusiasm, educational goals, methodological competence and knowledge a teacher affects on competence arise in their students, and because of that students results in mathematics also. Contemporary math education sets goals and models opposite than traditional teaching. Our research shows preference of classroom teachers to use contemporary or traditional concept of teaching mathematics. The results showed the most teachers have modern thinking about the mathematic teaching and a high level of awareness about the benefits and necessity of modernizing math teaching. Teacher's high level of awarnes toward contemporary approach we considered as good indicator of the progress of our teaching practice.

Keywords: math teaching, traditional concept of teaching, contemporary concept of teaching, primary teacher, preferences

Data processing and presentation in primary school at elementary level- Dealing with teaching experiences

Marija Pisk

Faculty of Education, University of Primorska, Koper, Slovenia

Abstract. Themes which, in the subject of mathematics, deal with information processing. Have been introduced into the syllabus in Slovenia during the transition to the nine-year schooling, in 1998. These themes have also remained a part of the syllabus after 2011, when the syllabi have been updated. The decision to introduce these themes was made with the purpose of increasing mathematical literacy and improve the quality of effective education regarding the processing, presentation, reading and interpretation of collected data, all in connection with an introduction to statistics. Pupils tackle their first cases in their pre-school period, later they upgrade their knowledge in school every year. They come from simple demonstrations introduced by professional staff in schools, to concrete, through graphic to a symbolic level. Here they consider the strategy of solving mathematical problems and the associated solving phases. In the 1st and 2nd grades they start with simple presentations and data reading with demonstrations using columns and lines. The figure presentation offers them a transition from the concrete to the graphic level. In 3rd grade they pass over to independent collecting, presentation, reading and interpretation of data. They also become used to employing various ways of data collection. In the 4th and 5th grades they can already use the gathered knowledge and independently plan research. In this paper you can find descriptions of cases which indicate the didactic manner of introducing any presentation. Difficulty gradation from one class to the other has also been indicated. These are themes which make classes more interesting and offer many sensible possibilities for connections with other subjects. The activities have been planned in such a way that they guide the pupil through the experience. This manner of operation offers to the pupils a sensible path to knowledge improvement, learning the skill of orientation in a presentation and gaining the skill of the reading

of data from a presentation. The introduction of these themes can also be executed through ICT. We can manufacture the material ourselves or look for them in cases available on the world wide web.

Keywords: Mathematical literacy, information processing, presentations, problem solving strategies, didactic path

Open-ended mathematical tasks for primary school youngsters

Diana Mesić[1] and Sanja Basta[2]

[1]Primary school Viktorovac, Sisak, Croatia
[2]Primary school 22. lipnja, Sisak, Croatia

Abstract. The school curriculum emphasizes that maths should be a practical and useful subject which students can understand and apply in everyday life to solve various problems. Particular goals of the national curriculum are aimed at developing students' competencies. Mathematical competence will enable a student to develop and apply mathematical thinking in solving simple problems. Students should realize the importance of maths during their education because it develops their awareness and understanding of the environments in which they live. It should enable pupils to manage risk and cope with change and adversity. PISA, being an important aspect of mathematical literacy, emphasizes that Maths should promote pupils' social and personal lives. It improves their schooling and teach them how to spend their free time in a more useful way. We all know that Croatian fifteen-year-old students did badly in mathematics last year. It was due to a traditional way of teaching Maths in our schools, the use of boring teaching methods and strategies where a set of exercises are done in a non-creative way (while according to PISA assessment items include a combination of multiple choice and open-ended tasks).

The teaching of mathematics is defined by the Curriculum which has strictly separated teaching themes, key terms and achievements, where more attention is paid to the realisation of the curriculum and not to the application of knowledge. Students usually learn everything by heart and cannot apply these competencies to real-world contexts. Being teachers practitioners we have noticed that teaching in a creative way is much more efficient (instead of using traditional methods). We apply various new methods while explaining a new subject matter.We have to use different types of exercises as means of stimulating and encouraging the best possible progress for all pupils. Maths should contribute to the

development of pupils' sense of identity through knowledge of the social heritage of diverse society and of the local, national and global dimensions of their lives.

Examples of open-ended and problem -solving tasks will be shown in this paper. These exercises require an integration of data while solving problems (which we try to incorporate into teaching). Primary-school teachers, living in the town of Sisak, should take into consideration this kind of teaching.

Keywords: mathematical competence, mathematical education, mathematical functionality, mathematical tasks, role of a teacher

Educational effects of the interactive method in studying the Pythagorean theorem in the elementary school

Sead Rešić[1] and Zemina Seferović[2]

[1]Faculty of Science, Department of Mathematics, University of Tuzla, Bosnia and Herzegovina
[2] Combined secondary school „Travnik“ in Travnik, Bosnia and Herzegovina

Abstract. The purpose of this paper was to show that traditional methods of teaching mathematics in elementary school do not produce satisfactory effects. The implementation of contemporary teaching methods, such as the interactive method, aims at maximum involvement of students, by means of which the passive listener turns into an active participant of teacher-student, teacher-group and student-group communication. The effects of interactive education methods are much stronger than those of the teacher-fronted instruction, and that is the goal of modern teaching.

Keywords: educational effects, interactive method, the Pythagorean Theorem

Fibonacci numbers and the golden ratio

Sead Rešić[1] and Senada Sofić[2]

[1]Faculty of Science, Department of Mathematics, University of Tuzla, Bosnia and Herzegovina
[2]Grammar school "Ismet Mujezinović", Tuzla, Bosnia and Herzegovina

Abstract. Mathematics is all around us and we help students to better understand the significance and importance of Mathematics as a science through correlating mathematics with other subjects, and thus accomplishing higher efficiency and economics in the lectures. Some of the subject units that give us the possibility to do so are Fibonacci numbers and the golden ratio. By relating the contents of the mathematics lectures with biology, art and informatics, it has been possible to encourage students to become interested and gain new knowledge in the world of mathematics through a certain activity. Scientists have always been interested in different kinds of regularities and appearance of numbers in nature. This kind of work contains three very interesting and related subjects: Fibonacci sequence, Pascal's triangle and the golden ratio, i.e. number Φ.

Fibonacci numbers are the kind of recursive sequence in which the sum of two precedent numbers gives the next. Leonardo Fibonacci reached this interesting number sequence while he was exploring the laws of breeding in rabbits. Flowers of many plants that create spiral structure have the number of petals that meets the certain number in Fibonacci sequence.

Second part of the work describes useful applications of Pascal's triangle and relation to Fibonacci sequence. Diagonal summing of the numbers from Pascal's triangle gives the Fibonacci sequence.

Third part of the work explores the presence of the golden number in ourselves and all around us. Leonardo Da Vinci performed a complete study of the human body and showed that its different parts are in proportion to golden ratio. Students (through group work or project) can determine by themselves through measurements which of those parts of body are in so-called divine proportion. We find Φ in everyday life, design, construction, esthetics, economics, physics,

mathematics and the space. The great pyramid, Parthenon, the church of Notre Dame, Fibonacci sequence, Da Vinci painting – The Last Supper are one of the examples that contain the length that are in scale close to golden ratio. We observe the world all the time and make decisions on the basis of what we see, so that children can better understand and remember if images are included in the process of studying. Construction of the golden ratio is done in the program GeoGebra which allows animation and leaves the traces upon construction.

Keywords: Fibonacci sequence, Pascal's triangle, golden ratio

Touch Math

Aleksandra Krampač-Grljušić, Ivana Pavleković and Monika Mihaljević

Elementary school Ljudevit Gaj, Osijek, Croatia

Abstract. Current legal documents of a large number of European countries give advantage to educating children who belong to the same generation but have different levels of knowledge skills and abilities. At the same time we are witnessing a large number of exclusion of students from schools and the low achievements of a significant number of students in Europe (EC, 2001, according to Bartolo et al., 2007). Mathematics learning disorders are noticed in achievements that decline and are lower than expected considering age, education and intelligence level. In regular classes there is a strong requirement for adjustment and preparation for teaching mathematics, not only for the students with disabilities, but for all students. "Touch Math" is an innovative teaching method that enhances mathematical achievements in the field of summing, subtraction, multiplication and division. The "Touch Math" program ensures multisensory approach and gradual instructions that apply dot notation for each number (not the numerical line) and that can be used by students with difficulties remembering facts. Benefits of multisensory methods in teaching basic concepts in mathematics are confirmed by many studies (Scott, 1993, Thornton, Jones and Toohey 1983, according to Calik and Kargin, 2010).

Teaching in accordance with the method of the "Touch Math", based on the direct teaching approach, is effective and sustainable. It can be generalized and it has a social value in teaching the basic summing skills to students with disabilities.

Keywords: addition strategies, touch math, students with difficulties

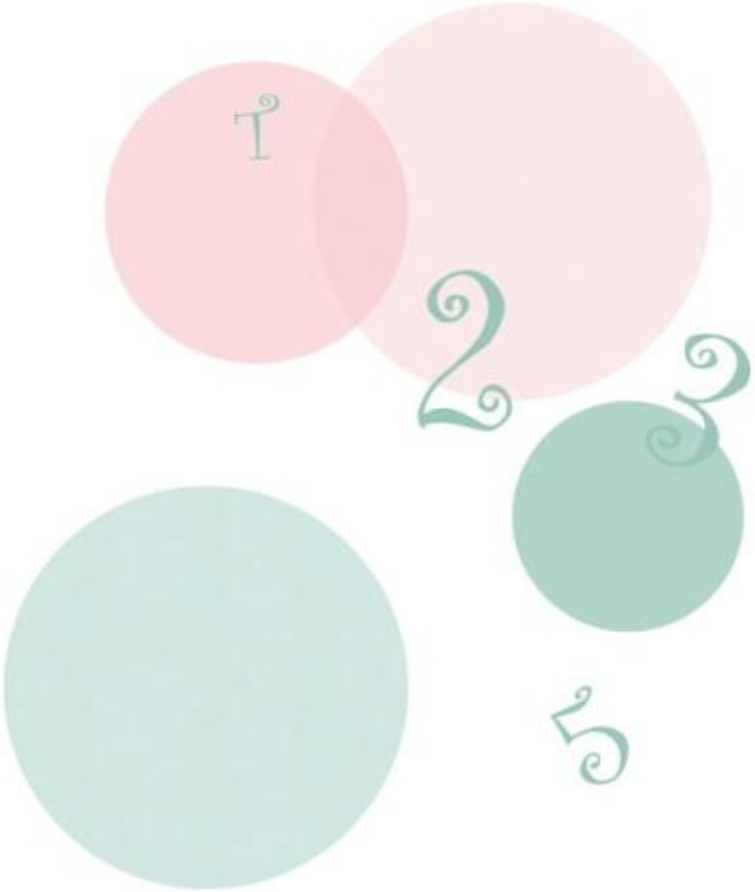
Visualization of mathematical concepts – some examples from geometry teaching

Katalin Munkácsy

ELTE, Budapest, Hungary

Abstract. The abstract nature of mathematical concepts requires more effective illustration. For beginners this is solved with difficulty because of good tools, e.g. Dienes set often does not concretize the concepts, but it also represents a higher level of abstraction. Children's use of everyday objects can help implement the transition between reality and abstraction. Examples of this process will be shown in this paper.

Keywords: concept teaching, teaching of disjunction, teaching of the concept of the plane



ISBN 978-953-6965-30-4